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Determining High School Students' Mathematical Thinking Styles: Latent Class Analysis

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Abstract

The purpose of this study is to adapt the Mathematical Thinking Style (MTS) scale to Turkish, and to examine mathematical thinking styles of students by gender. The MTS scale was developed by Rita Borromeo Ferri at Kassel University in the scope of her 2011/2012 project, "Mathematical Thinking Styles in School and Across Culture (MaTHSCu)." With this scale, it can be determined which integrated, visual/pictorial, and analytic/formal thinking styles are preferred by students. After examining the psychometric structure of the Turkish form of the scale, it has been shown that mathematical thinking styles can be determined by latent class analysis (LCA). The study was comprised of 336 participants studying in Grades 9 and 10 in Ankara. The construct validity of the scale was examined by factor analysis, and the adaptive scale was found to have the same structure as the original scale. As a result of the LCA, it has been determined that students can be divided into three homogeneous groups with high classification reliability in compliance with the theoretical structure. The study found that the students mostly have an integrated thinking style. Female students had a more analytic thinking style compared to boys, and male students had a more visual thinking style compared to girls. In addition, the findings show that the MTS scale is a valid and reliable measuring instrument for the Turkish language, and that the MTS of students can be determined using LCA with a high level of confidence. The determination of students' mathematical thinking styles by LCA to be within the bounds of possibility constitutes the original contribution of this study to the literature.

Keywords

Thinking style
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Introduction

Thinking is a phenomenon that is an essential part of what it means to be human, and its importance within educational fields is increasing day by day (Mubark, 2005). According to Yıldırım (2015), who defines "thinking" as an activity to solve a problem or trouble, thinking is a process that reveals understanding at the same time. If the thinking process results from mathematical studies or mathematical situations, it is defined as mathematical thinking (Burton, 1999). Likewise, Henderson et al. (2003) define mathematical thinking as thinking that uses mathematical techniques, concepts, and processes either directly or indirectly when solving problems. According to these viewpoints, it can be said that every problem-solving activity requires mathematical thinking. It is according to this perspective that mathematical thinking is discussed in this study. However, in the problem-solving process, individual differences can lead to the use of different approaches in mathematical thinking. In this study, the different approaches that individuals use in their mathematical thinking processes are considered as mathematical thinking styles. On the other hand, studies on mathematical thinking styles were conducted using the measurement tools developed by Suwarsono (1982) and Presmeg (1985). However, the usefulness of these measuring tools is decreased because of the time required to apply them. In addition, the absence of a statistically based cut-off scoring value when determining the mathematical thinking styles of students using these measurement tools may cause some inaccurate inferences. Because of this, the present study was conducted with the Mathematical Thinking Style (MTS) scale based on Borromeo Ferri and Kaiser's (2003) theory of mathematical thinking styles. With the data obtained from this scale, students' mathematical thinking styles can be determined based on probability by latent class analysis (LCA), which is an individual-based technique.

Thinking Styles

Why do some students with similar abilities succeed, while others fail? There may be more than one answer to this question when considering the many factors that influence learning. Educators examine the influence of teaching methods and characteristics that affect learning, in improving students' achievements in general (Zhang, 2001). Nevertheless, it is seen that some studies have also been conducted to examine the influence of thinking styles on the learning of students (e.g., Zhang, 2001, 2004). Variables such as thinking styles that are beyond students' abilities to control also play a significant role in the students' performance at school and in their interaction with their teachers (Sternberg & Grigorenko, 1995). Even though student intelligence is usually determined by the abilities they demonstrate, students' performance does not depend solely on their abilities. Besides ability, the style known as "the tendency to use the ability" may also make a difference in students' performance (Sternberg & Grigorenko, 1995; Kılıç, 2017). Zhang (2001), in his study conducted in China and Hong Kong with university students, reached the conclusion that thinking styles can forecast academic achievement. However, this study found that the styles that are effective in students' achievement may not show similar effects in other countries because thinking styles can be influenced by culture, and differences in thinking styles of individuals in different cultures can have different effects in the learning process (Zhang, 2001). Therefore, the effect of studies on thinking styles needs to be examined for each culture.

Mathematical Thinking Styles

Mathematical thinking styles are not about how well one understands mathematics, but only reflect one's approach in the process of understanding and learning mathematics (Borromeo Ferri, 2012, 2015). Borromeo Ferri (2015) bases her definition of mathematical thinking style on Sternberg's (1997) thinking styles definition. According to Sternberg (1997), thinking style is not a talent, but a way of thinking—the path that is chosen by individuals through which to use their abilities. Similarly, Borromeo Ferri (2015), as mentioned above, defines mathematical thinking styles as one's approach or way to learn and understand mathematics. In the literature, there are many classifications related to the mathematical thinking style. Krutetskii (1976) mentions three types of mathematical thinking styles: analytic/formal, geometric, and harmonic. Analytic thinkers prefer verbal logical solutions instead of visual solutions, whereas geometric thinkers prefer visual solutions, and harmonic thinkers prefer either visual or analytic thinking. Suwarsono (1982) and Presmeg (1986) state that there are two types of mathematical thinking styles, visual and non-visual. Another researcher, Burton (1999), splits thinking styles into three categories: visual, analytic, and conceptual. In general, she classifies those who think with pictures (drawings, graphics, etc.) as being visually oriented; those who think with symbols and rules as being analytically oriented; and those who classify and think with ideas as being conceptually oriented (see Burton 1999). When the relevant literature is examined, Krutetskii (1976) mentions harmonic thinking as well as analytical and geometric thinking styles, while Burton (1999) mentions conceptual thinking alongside analytical and visual thinking styles. The first two thinking styles in Krutetskii (1976) and Burton's (1999) classifications seem to be similar, although there are differences in their nomenclature. However, Krutetskii's (1976) harmonic thinking style and Burton's (1999) conceptual thinking style are different from each other, and it seems that there is no complete consensus on them by scholars. One reason for this can be that these classifications on mathematical thinking styles are usually based on observations, not on empirical studies. Borromeo Ferri and Kaiser (2003) carried out an empirical study with students in Grades 9 and 10 to determine thinking styles using grounded theory. Borromeo Ferri and Kaiser (2003) found that that conceptual thinking could not be validated in that empirical study because conceptual thinking is a higher level of thinking that may be more appropriate for university-level students. In addition, they observed that students sometimes think visually and sometimes analytically, depending on the question. They classified such thinkers as integrated thinkers (Borromeo Ferri & Kaiser, 2003; Borromeo Ferri, 2012, 2015). As a result, Borromeo Ferri (2012, 2015) identified three types of thinking styles: visual, analytic, and integrated, depending on the empirical study they used with students in Grades 9 and 10. This classification is similar to the classification made by Krutetskii (1976). However, Borromeo Ferri (2015) introduced a more detailed functional definition of the mathematical thinking style to the literature. In this context, Borromeo Ferri (2015) describes visual, analytic, and integrated thinking as follows:

> Visual thinking style: Visual thinkers show preferences for distinctive internal pictorial imaginations and externalized pictorial representations as well as preferences for the understanding of mathematical facts and connections through holistic representations. The internal imaginations are mainly affected by strong associations with experienced situations.
>
> Analytical thinking style: Analytic thinkers show preferences for internal formal imaginations and for externalized formal representations. They are able to comprehend mathematical facts preferably through existing symbolic or verbal representations and prefer to proceed rather in a sequence of steps.
>
> Integrated thinking style: These persons combine visual and analytic ways of thinking and are able to switch flexibly between different representations or ways of proceeding. (p. 105)

Example solutions of mathematical thinking styles proposed by Borromeo Ferri (2015) are given in Figure 1.

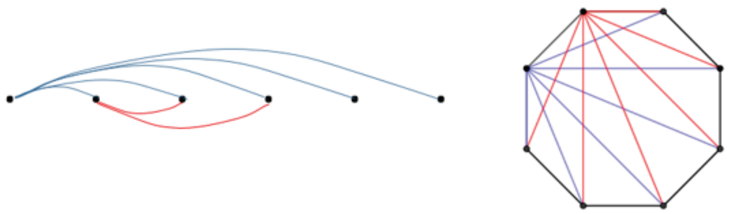
Birthday party Eight persons are gathering at a birthday party. Everybody wants to clink his glass exactly once with each other. How often will the glasses be clinked?	
Analytic solution	Visual solution
7+6+5+4+3+2+1=28 $\frac{7.8}{2}=28$	

Figure 1. Example Solutions of Mathematical Thinking Styles

The literature involves studies on the analysis of mathematical thinking styles using the "Mathematical Processing Instrument" developed by Suwarsono (1982) (see Coskun, 2011; Hacıömeroğlu & Hacıömeroğlu, 2013; Mainali, 2014), and there have also been studies conducted by the "Mathematical Processing Instrument" developed by Presmeg (1985) (see Galindo-Morales, 1994; Sevimli, 2013; Taşova, 2011). However, the fact that the measuring instruments developed by Suwarsono (1982) and Presmeg (1985) take a long time to use reduces their usefulness. In addition, different approaches have been used in the scoring of these instruments. As stated by Lean and Clements (1981), Suwarsono (1982) gave +2 points to the correct visual solution, +1 point to the incorrect visual solution, 0 point to the unanswered question, -1 point to the incorrect nonvisual solution, and -2 points to the correct nonvisual solution in the pilot study of his own thesis. Later, Suwarsono changed the scoring system to be 2 points to the correct or incorrect visual solution, 1 point to the solution where the style could not be determined or to unanswered question, and 0 to the non-visual solution, due to the criticisms against the scoring system, which is based on the accuracy of solutions (Suwarsono, 1982). Another researcher, Mainali (2014), performed a scoring of Suwarsono's (1982) scale by giving +1 point to the visual solution, -1 point to the analytical solution, and 0 points to the blank solution. The similar situation is observed in other studies as well (e.g., Coskun, 2011; Lean & Clements, 1981). On the other hand, it is also stated that the difference in scoring does not make a statistically significant difference (Suwarsono, 1982). In this way, two different mathematical thinking styles—visual and non-visual—can be determined. In subsequent studies (e.g., Coskun, 2011; Galindo-Morales, 1994; Sevimli, 2013; Taşova, 2011), students were distinguished according to three different mathematical thinking styles. In addition, Galindo-Morales (1994) classified 15% of students who are in the upper group as being visual thinkers, 15% of students who are in the lower group as being non-visual thinkers, and the other students as being harmonic thinkers, according to their total test score. These scoring systems are not based on statistics. In addition, the lack of a statistically determined cut-off value to determine the mathematical thinking style of students in using these measuring instruments may affect the reliability of the classification obtained through them. In fact, Sevimli (2013) and Taşova (2011) state that students' mathematical thinking styles cannot be determined precisely by considering their cognitive preferences in the problem-solving process. Therefore, it may be considered more appropriate to determine the mathematical thinking styles within the bounds of possibility. This is because visual and non-visual mathematical thinking styles can be determined through the scoring method of Suwarsono (1982) and Presmeg (1985), but individuals who use visual and non-visual elements together cannot be determined. Suwarsono (1982) stated that theoretically it was logical that the students should be divided into three groups according to their mathematical thinking styles, but due to the uncertainty of cut-off values, she preferred to divide the students into two groups. Furthermore, in the method used by Galindo-Morales (1994), there is no statistical basis for the cut-off values (15%). This may lead to several inaccurate inferences in the determination of mathematical thinking style. For this reason, the need arose for a measuring instrument that would both classify mathematical thinking styles within the bounds of possibility and measure them in a shorter time. Borromeo Ferri (2012) improved the mathematical thinking style scale; with it, students' choice of visual, analytic, and integrated thinking styles can be determined by LCA, which is an individual-based technique. Through LCA, it can be determined which thinking style students have within the bounds of possibility, and the classification reliability (entropy) can also be calculated. However, the measuring instruments developed by Suwarsono (1982) and Presmeg (1985) do not have such a classification reliability. From this point of view, it is hoped that the present study will shape future studies to be carried out on thinking styles.

Purpose and Importance of the Study

There are gender-related problems in mathematics education, and these problems vary according to social contexts and cultures (Wedege, 2011). In particular, these problems arise in students' mathematical performance. The results of the international examinations show that gender creates different results in student performances according to different cultures (Organization for Economic Co-operation and Development [OECD], 2016). In the literature, there are studies showing that gender makes a difference in student performances (e.g., Işıksal & Çakıroğlu, 2008; Lane, Wang, & Magone, 1996; Lyons-Thomas, Sandilands, & Ercikan, 2014), as well as studies showing that gender does not make a difference (e.g., Işıksal & Aşkar, 2005). Lyons-Thomas et al. (2014) in their study found that male students compared to female students in Canada, China (Shanghai), Finland, and Turkey perform better in mathematics, but the performance varies according to gender in different test items. In another study, Lane et al. (1996) found that male students were good at questions that included shapes and geometry, as compared to female students. In addition, according to a study conducted in Turkey, male students perform better in geometry compared to female students, while female students perform better in symbolic operations (e.g., Işıksal & Çakıroğlu, 2008). Similar findings were found in the study of Garner and Engelhard (1999). Abedalaziz (2010) also found that male students outperformed females in geometry and real-world problems. However, these studies are done merely to determine the existence of gender differences in student performances, and they are insufficient to explain the reasons there are gender differences in student performance.

Students' mathematical thinking may play a role in math performance because in the mathematical thinking process, students may display different approaches due to their mathematical thinking styles (Borromeo Ferri, 2004, as cited in Blum & Borromeo Ferri, 2009). This is not related to how well a person understands mathematics, but it reflects his or her approach in the process of understanding and learning mathematics (Borromeo Ferri, 2012, 2015; Burton, 1999; Sternberg, 1997). In this study, the theoretical framework and mathematical thinking style scale of Borromeo Ferri (2012, 2015) was used, as it was built on a more stable basis than Burton's (1999), which was based on observations only. Furthermore, considering the mathematical thinking theory of Borromeo Ferri (2012, 2015), it is not known which mathematical thinking styles are dominant in the Turkey student sample. On the other hand, the mathematical thinking styles of students lets their teachers know their preferred learning style for mathematics. Students understand mathematics better if teachers can teach them according to students' own learning styles, because variables such as thinking styles, which are beyond students' ability to control, play a significant role in students' performance at school and in their interaction with teachers (Sternberg & Grigorenko, 1995). Although there are studies in the literature that examine students' thinking styles by gender (Piaw, 2014; Wang & Tseng, 2015; Zhang, 2010), there are few studies, especially in Turkey, that examine mathematical thinking styles by gender (Dede, Akçakın, & Kaya, 2017). Dede et al. (2017) determined the pre-service teachers' mathematical thinking styles with open-ended questions, and found that male students prefer more visual thinking styles than female students. In addition, especially there has been no studies on the mathematical thinking styles of high school students in Turkey. Thus, this study is expected to create a basis for future research.

In this context, the purpose of this study is to adapt the MTS scale to Turkish, and to examine the mathematical thinking styles of students by gender. After examining the psychometric structure of the Turkish form of the scale, it has been shown how mathematical thinking styles can be determined by LCA. From this point of view, it is hoped that this study will guide researchers in using LCA when determining the mathematical thinking styles of individuals. Unlike other measuring instruments for mathematical thinking styles (Presmeg, 1985; Suwarsono, 1982), adapting the MTS scale to Turkish can collect data from a larger mass of people in a short time, and mathematical thinking styles of students can be determined quickly and easily. Considering all these, this study seeks an answer to the following research questions:

1. What are the mathematical thinking styles of students?
2. How do mathematical thinking styles of the students vary by gender?

Method

Model of the Research

In this study, students' mathematical thinking styles were collected from participants on a volunteer basis through the MTS scale developed by Borromeo Ferri (2012). The design of the study was a cross-sectional survey model. In this model, the data are gathered at a single time from the sample (Fraenkel, Wallen, & Hyun, 2012).

Participants

The study was composed of 336 participants studying in Grades 9 and 10 in Ankara. A total of 224 (66.67%) students, 128 (57.1%) female and 96 (42.9%) male students, were studying in Grade 9, and a total of 112 (33.33%) students, 56 (50%) female and 56 (50%) male students, were studying in Grade 10. The data was collected from students from Grades 9 and 10 studying in three schools who were selected in accordance with the random cluster sample method on a volunteer basis.

Data Collection Tool

Required permissions were obtained from Rita Borromeo Ferri, the developer of the scale, to adapt the MTS scale to Turkish. Through this scale, it can be determined which integrated, visual/pictorial and analytic/formal thinking styles are preferred by students. The scale consists of two parts. The first part includes questions about demographic characteristics such as gender, age, and grade level. This study does not include any question that reveal the identities of the participants due to ethical principles. The MTS scale was developed by Borromeo Ferri (2012) at Kassel University within the scope of the MaTHSCu project. The scale consists of four-point Likert-type items, and these items are scored as "strongly agree (1)," "agree (2)," "disagree (3)," and "strongly disagree (4)." The scale consists of 11 items in total: five items are about the analytic thinking, five items are about the visual thinking, and one item is an objective item. Borromeo Ferri (2012, 2015) noted that in her last pilot study, the Cronbach's alpha reliability was found to be .77 for the visual dimension and .90 for the analytic dimension. The scale includes items related to analytical thinking such as "I like to use a formula when I have to solve a mathematical problem," and those related to visual thinking such as "Visual images are helpful for me to understand mathematics."

Analysis of Data and Determining Mathematical Thinking Styles

The reliability of the scale was determined by the Cronbach's alpha internal consistency coefficient and McDonald's omega reliability coefficient, since the scale is congeneric, as Cronbach's alpha indicates the lower limit of the reliability in congeneric measurements (Yurdugül, 2006). For McDonald's omega reliability coefficient, the nonstandardized factor loads of the confirmatory factor analysis conducted by releasing parameters are needed. Factor structure of the scale was examined by exploratory factor analysis (EFA) and confirmatory factor analysis (CFA). What kind of mathematical thinking styles individuals have is determined by the latent class analysis (LCA) technique, depending on the scores obtained by the scale. LCA is a statistical technique that classifies individuals into homogeneous groups (Geiser, 2013). Latent variables and indicators are at a categorical level. LCA is like factor analysis, but similar items come together in factor analysis, whereas similar individuals come together in LCA. On the other hand, in LCA, parsimony and explicability are important in choosing the model (Collins & Lanza, 2010). To determine the most appropriate class for LCA, a series of increasing number LCA models are tested, and each model is compared with the previous model (Wang & Wang, 2012). Relative model fit techniques such as the Bootstrap Likelihood Ratio Test, the Vuong-Lo-Mendell-Rubin Test, and the Lo-Mendell-Rubin Adjusted LRT test are included in model comparison. In addition, such information criteria as the Akaike information criterion (AIC) (Akaike, 1987) and the Bayes information criterion (BIC) (Schwarz, 1978) are also used. In general, the BIC value is used when determining the LCA model, and the lowest BIC value indicates the most suitable model (Magidson & Vermunt, 2004; Oberski, 2016). After determining the most appropriate model, the classification reliability is examined and the existing classes are named, and finally class estimations are done for individuals (Wang & Wang, 2012). Classification reliability is measured by entropy value (Celeux & Soromenho, 1996). The entropy value ranges between 0 and 1, and the values which are close to 1 indicate a good classification (Geiser, 2013).

Krutetskii (1976) states that there is no definite limit value in distinguishing between these styles, except extreme values, as is the case with any psychological typology, and there are transitional variants between styles. From this point of view, it may be more appropriate to determine the styles within the bounds of possibility. The mathematical thinking styles of students can be determined within the bounds of possibility. One point is given for statements with which students agree or strongly agree, and zero points are given for statements with which students disagree or strongly disagree (Geiser, 2013). In this way, two categories of data are obtained to perform the LCA analysis.

Results

The study's findings have been addressed in two main topics in compliance with the purpose of the research. First, the suitability of the scale to the Turkish language was examined, and then the theoretical structure was examined by LCA. Finally, the mathematical thinking styles of the students were determined.

Examination of the Psychometric Properties of MTS Scale

In adapting the MTS scale to Turkish, the psychometric structure of the Turkish form of the scale was examined first.

Reliability of the MTS Scale

The reliability of the MTS scale was checked with the McDonald's omega and Cronbach's alpha reliability coefficients. In addition, it is recommended to examine McDonald's omega reliability as well in congeneric measurements (Yurdugül, 2006). According to the analysis, Cronbach's alpha reliability was found to be 0.74 and McDonald's omega reliability was found to be 0.75 for the visual dimension of MTS scale. For the analytic dimension of the MTS scale, Cronbach's alpha reliability was found to be

0.86 and McDonald's omega reliability was found to be 0.86. As these values were higher than the critical value of 0.7, which has been determined for reliability (Nunnally, 1978), the scale was judged reliable.

Validity of the MTS Scale

The findings on the language and structure validity of the scale are addressed in this section.

Language Validity

The advanced translation technique was used when adapting the scale to Turkish (Hambleton, 2005), as it provides stronger proofs in equalizing tests within the source and the target language (Hambleton & Patsula, 1998). In this process, a five-step translation, review, adjudication, pretesting, and documentation (TRAPD) team translation model proposed by Harkness, Villar, and Edwards (2010) was used. In this technique, at least two translators translate the measuring instrument into the target language independently (translation). In the second stage, the translations in the source and the target language are reviewed by at least one proofreader and suggestions are made (review). In the third stage, the translation suggestions to be used are decided with at least one referee and the translators (adjudication). At the end of these processes, the measuring instrument becomes ready for pilot study (pre-test). Procedures in these processes are recorded for re-examination when necessary (documentation).

In addition to the TRAPD team translation model, whether the materials in this study are understandable in terms of language has also been examined by a specialist in the field of Turkish education. In addition, prior to the pilot study, the items of the scale were examined in terms of understandability by applying them to four students from Grades 9 and 10. At the end of this process, the question of whether the scale obtained is a valid and reliable measuring instrument for the target language, Turkish, was examined with the structure validity.

Construct Validity

Principal component analysis was performed through a varimax rotation procedure to determine the structural validity of the MTS scale. The suitability of the data for the factor analysis was examined by the Kaiser-Meyer-Olkin measure and the Bartlett test of sphericity. According to Kaiser (1974), the fact that this value is higher than .8 is considered quite good for the sample size. This value is .81 for the MTS scale, and the sample size was found suitable for the factor analysis. For the factor analysis, there should be a correlation between items; in other words, the correlation matrix obtained from the dataset must be different from the unit matrix. The significance of the Bartlett test (p <.05) indicates that this correlation matrix is different from the unit matrix (Field, 2009). This value is found to be significant according to the analysis results ($\chi 2$ (45) = 1194.937, p <.05). This result shows that there is a correlation between the items, and that the correlation matrix is different from the unit matrix. This indicates that items can form factors (Field, 2009). Also, the data should not be related to other data. In the factor analysis, if the determinant of the correlation matrix is higher than 0.00001, it means there is no multi-connectivity problem (Field, 2009). The determinant of the correlation matrix was found to be 0.027 because of the analysis. The results of factor analysis are given in Table 1.

Table 1. Factor Structure of the MTS Scale

		EFA Varimax Rotation		CFA Nonstandardized			
		Factor Loads		λ			
Items	**Factors**	**Visual**	**Analytic**	**Visual**	**Analytic**	σ^2 *(E)*	R^2
q1mdv1	Visual	**.830**	-.033	**0.41**		0.32	0.34
q1mdv2	Visual	**.875**	-.110	**0.53**		0.45	0.39
q1mdv3	Visual	**.825**	.008	**0.27**		0.60	0.11
q1mdv4	Visual	**.733**	.015	**0.56**		0.35	0.47
q1mdv5	Visual	**.765**	-.011	**0.63**		0.22	0.64
q1d		Neutral Item					
q1mdf1	Analytic	-.010	**.709**		**0.67**	0.26	0.63
q1mdf2	Analytic	-.092	**.748**		**0.78**	0.17	0.79
q1mdf3	Analytic	.047	**.481**		**0.64**	0.30	0.58
q1mdf4	Analytic	-.086	**.739**		**0.52**	0.42	0.39
q1mdf5	Analytic	.006	**.812**		**0.66**	0.54	0.45
Eigenvalue		3.34	2.45				
Variance		32.76	25.12				
Total Variance		57.88					

Table 1 shows that the factor loads of the items in the visually named factor vary between .830 and .765, and the factor loads of the items in the analytically named factor vary between .812 and .481. Also, the eigenvalue of the visual factor is 3.34, while the eigenvalue of the analytic factor is 2.45. The visual factor itself accounts for 32.76% of the total variance, the analytic factor accounts for 25.12% of the total variance, and the whole scale accounts for 57.88% of the total variance. This finding indicates that the scale can be measured with a high variance ratio. As a result, two factors have been found, the eigenvalue of which is higher than 1 according to the factor determination criterion (Kaiser, 1960). Confirmatory factor analysis was done after the explanatory factor analysis.

The chi-square value was found significant in the analysis ($\chi 2$ (34) = 70.90, p <.05). But this is expected when the sample is bigger. In this case, it is recommended to look at the proportion of the chi-square value to the degree of freedom, and this value ($\chi 2$ / sd) was found to be 2.09. This shows that the model data fit is excellent (Kline, 1998; Çokluk, Şekercioğlu, & Büyüköztürk, 2010). On the other hand, the fact that the comparative fit index (CFI) and the normed fit index (NFI) values are over .95, the standardized root mean square residual (SRMR) value is below .05, and that the adjusted goodness of fit index (AGFI) value is over .90, shows that the model and data fit is high (Hu & Bentler, 1999). Hair Jr., Black, Babin, and Anderson (2014) accept that the model data fit is good if the root mean square error of approximation (RMSEA) value is lower than .07, provided that the CFI value is greater than .97 when the sample number is higher than 250 and the number of items is below 12. In CFA, it has been found that CFI = 0.98, NFI = 0.95, AGFI = 0.93, SRMR = 0.041 and RMSEA = 0.057 (90% confidence interval [CI] = .038–.076). All these findings indicate that model data fit of the MTS scale is good. As a result, if the values of the model data fit are examined, the model can be said to have a good fit. The model for confirmatory factor analysis is shown in Figure 2.

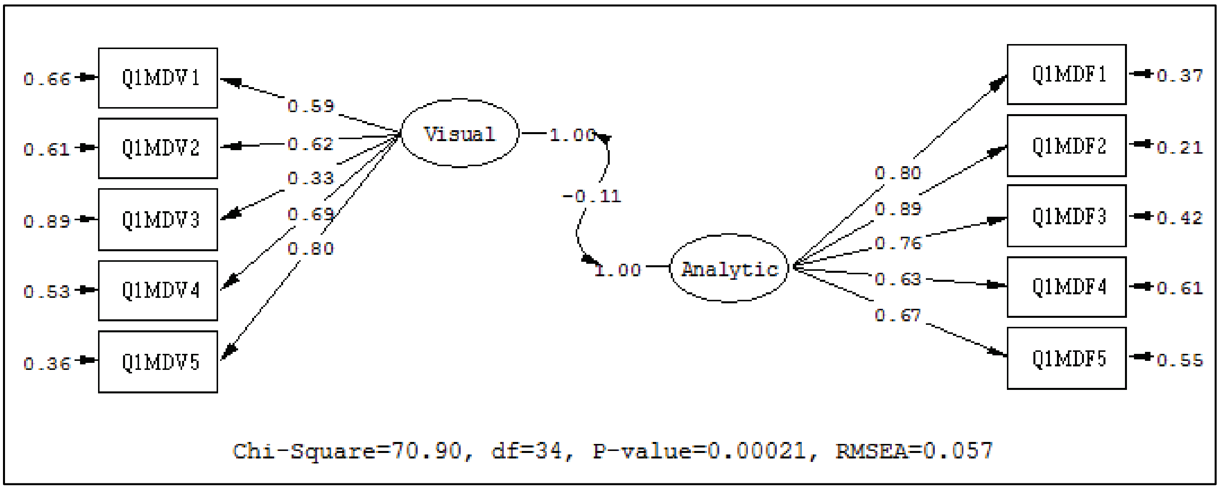


Figure 2. Confirmatory Factor Analysis Diagram for the MTS Scale

Determining Mathematical Thinking Styles of Students by the LCA

To determine the mathematical thinking styles of the students, LCA was conducted, and sequential model comparison analysis was carried out for Classes 1, 2, 3, and 4. The analyses result can be seen in Table 2.

Table 2. Relative Model Fit Index and Information Criteria for the Mathematical Thinking Style Scale

Sequential model comparisons	**Result of MTS group comparisons (N=336)**			
	1-class LCA	**2-class LCA**	**3-class LCA**	**4-class LCA**
BIC	3734.986	3392.614	3308.166	3308.343
Vuong-Lo-Mendell-Rubin likelihood ratio p-value	_	<0.001	<0.001	0.0870
Lo-Mendell-Rubin Adjusted likelihood ratio test (LRT) *p*-value	_	<0.001	<0.001	0.0899
Parametric bootstrapped likelihood ratio test *p*-value	_	<0.001	<0.001	<0.001

_: Not applicable

For the selection of the best fitting model, several different criteria were developed. LCA was implemented with one, two, three, and four-class models. The statistical results of the Vuong-Lo-Mendell-Rubin test and the Lo-Mendell-Rubin adjusted LRT test support the three-class model. However, the parametric bootstrapped likelihood ratio does not support the three-class model. Besides, LCA models can also be compared using information criteria like AIC and BIC. Based on simulation studies, the BIC index is recommended as the best for determining the number of class comparison models in LCA (Nylund, Asparouhov, & Muthén, 2007; Geiser, 2013). As a result of the BIC analysis, the three-class model fits better than both two and four class-models. This means that statistical method confirms the theory of mathematical thinking style. According to statistical analysis and mathematical thinking style theory, LCA was carried out with the three-class model. The entropy value for the three-class model is found to be 0.822, thus we can say that the accuracy of the non-random classification probability is 82.2%. This result indicates a high entropy value as per Clark (2010). These results show that latent classification is done with high accuracy. The average latent class posterior probabilities for the most likely latent class membership are reported in Table 3.

Table 3. Average Latent Class Probabilities for Most Likely Latent Class Membership for MTS

	Probability of Visual MTS	Probability of Analytic MTS	Probability of Integrated MTS
1	**0.942**	0.013	0.045
2	0.027	**0.889**	0.084
3	0.036	0.027	**0.937**

According to Nagin (2005), average latent class probabilities for most likely latent class membership should be 0.70 or above. The probabilities for most likely latent class membership for students assigned to the first class was 0.942, second class was 0.889, and third class was 0.937. These results cover Nagin (2005)'s criterion for all groups.

The Mplus profile graph showing the conditional response probabilities for the second category of the MTS items in each of the three latent classes is given in Figure 3.

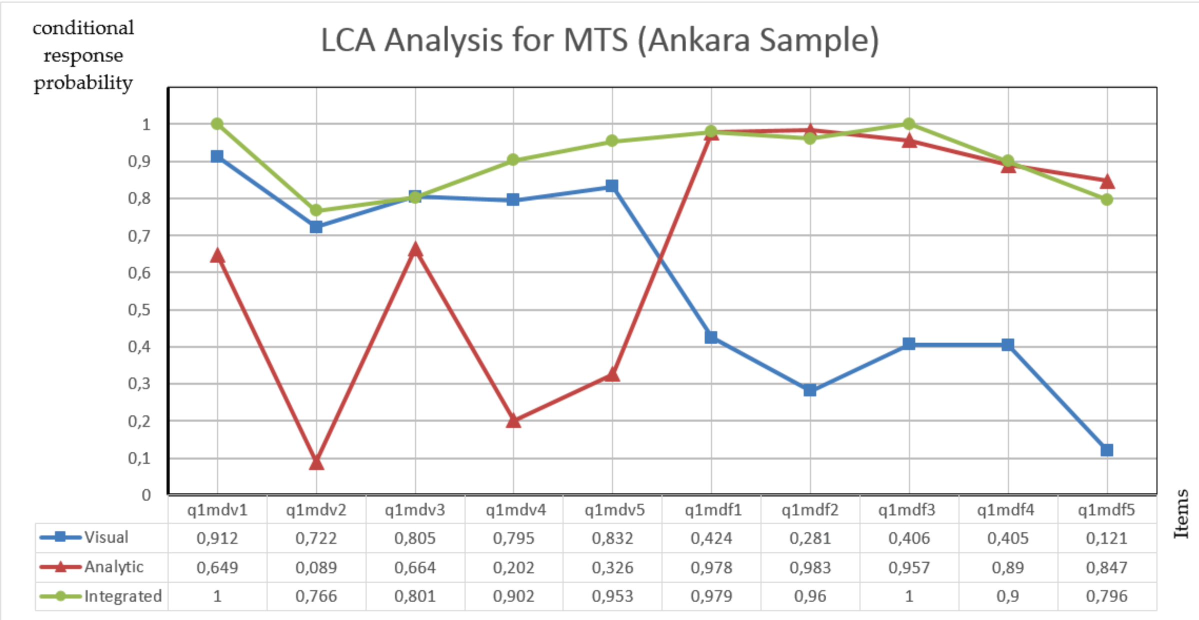


Figure 3. Mplus profile graph showing the conditional response probabilities for the second category of the MTS items in each of the three latent classes. The items are on the x-axis, whereas conditional probabilities of the ''strongly agree/agree'' category are placed on the y-axis.

Considering the Mplus profile graph, each category was named according to a mathematical thinking style theory. Thus, the green line ( ) was named as an integrated thinking style because probabilities were higher for all questions. The blue line ( ) was named as a visual thinking style because the visual questions' probability levels were higher than the analytic ones. The red line ( ) was named as an analytic thinking style because the analytic questions' probability levels were higher than the visual ones. According to the Mplus profile graph, analytical students tend to use their abilities in an analytic way, but they also prefer visual explanations from their teachers or by sketches on the board. Even though they do not prefer the visual thinking style, they like geometry because they think that a drawn figure can also be a possible solution.

Students were separated into three homogeneous groups by LCA with high classification. As a result of the LCA, number of students in each class was shown in Table 4.

Table 4. Number of Students in Each MTS Class

MTS Groups	Male		Female		Total	
	f	%	f	%	f	%
Visual thinking style	63	41.4	51	27.7	114	34
Analytic thinking style	22	14.5	42	22.8	64	19
Integrated thinking style	67	44.1	91	49.5	158	47
Total	152	45.2	184	54.8	336	100

Table 4 shows that most students have an integrated thinking style, and that fewest have am analytical thinking style. Of the male students, 63 (41.4%) had a visual thinking style, 22 (14.5%) had an analytic thinking style, and 67 (44.1%) had an integrated thinking style. Of the female students, 51 (27.7%) had a visual thinking style, 42 (22.8%) had an analytic thinking style, and 91 (49.5) had an integrated thinking style. On the other hand, boys tend to think more visually than girls, and girls tend to think more analytically than boys. Moreover, Table 4 also shows that girls tend to think more integratedly than boys.

Conclusion, Discussion, and Suggestions

In this study, the MTS scale was adapted to Turkish, and the mathematical thinking styles of 9th and 10th grade students were examined according to gender. For this purpose, the MTS scale developed by Rita Borromeo Ferri at Kassel University in the scope of her MaTHSCu project was adapted to Turkish using the TRAPD team translation model, which is an advanced translation technique. The reliability analysis determined that the data obtained for each sub-dimension was reliable because the Turkish form of the MTS scale was higher than 0.7 (Nunnally, 1978), which is the lower limit of reliability in the visual and analytic subdimensions. In addition, the factor structure of the MTS scale conforms well with the Turkish language. Thus, the Turkish version of the MTS scale is a reliable and valid measuring instrument. After examining the psychometric structure of the Turkish form of the scale, the mathematical thinking styles described by Borromeo Ferri (2015) were determined through LCA. As a result of the LCA, students were divided into three homogeneous groups with high classification reliability in conformity with the theoretical structure. The findings show that MTS scale is suitable for the Turkish language and that it can determine the mathematical thinking styles of students at a high entropy level.

Through this study, it is seen that the students from Grades 9 and 10 mostly have integrated thinking styles. Students who prefer this style can use visual and analytical thinking styles at the same time, or can switch between these styles (Borromeo Ferri, 2015). On the other hand, there are fewer students with the analytic thinking style, and female students have a higher level of analytic thinking style compared to boys. It is seen that male students are more likely to think visually compared to girls. Similar results were found in the study of Dede et al. (2017), where it was observed that male students tend to think more visually than female students; some studies indirectly support this result (Garner & Engelhard, 1999; Işıksal & Çakıroğlu, 2008). In the study of Işıksal and Çakıroğlu (2008), it was found that male students performed better in geometry than female students, while female students performed better in symbolic operations than male students. Considering that geometry requires visual thinking and symbolic operations require analytical thinking, the reason male students performed better in geometry than female students in the Işıksal and Çakıroğlu study (2008) might be that, as in this study, male students preferred more visual thinking styles than female students.

Similarly, it is conceivable that female students perform better in symbolic operations because of their tendency to use analytical thinking more than male students. In support of this, Kılıç (2017) states that mathematical thinking styles may affect students' performances. However, further research is needed to be able to say that this result stems from students' mathematical thinking styles.

In this study, mathematical thinking styles of students were determined on a probability basis in accordance with the thinking styles theory. Because the thinking styles of individuals may change over time, and individuals may behave somewhat flexibly in their use of these styles, these styles may change depending on the given situation. In other words, individuals' styles may vary from situation to situation (Zhang, 2001). Similarly, Krutetskii (1976) states that visual thinkers think analytically, even to a small extent, and likewise, analytic thinkers think visually, even to a small extent. In addition, integrated thinkers use an equal level of visual and analytic thinking. The graphic (see Figure 3) that emerged because of the LCA, as stated above, is consistent with Krutetskii's findings (1976) on mathematical thinking styles. In this respect, it is considered appropriate to determine the styles of individuals probabilistically. Therefore, the present study is believed to provide a rich basis for future studies.

The results obtained in this study provide information to mathematics teachers about mathematical thinking styles of students. By using the scale in this study, teachers can also determine their students' mathematical thinking styles and arrange their learning environments according to students' preferences, because there are different ways for teachers' styles to harmonize with students' needs, and students can learn better in learning environments that appeal to their own learning styles. At the same time, teachers may perceive students who match with their own style as being more successful, so they may value them more (Zhang & Sternberg, 2001); teachers may also overestimate such students (Sternberg & Grigorenko, 1995). In this respect, teaching by considering the mathematical thinking styles of one's students may enable them to learn better.

Since there are cultural differences in thinking styles, differences in the thinking style of individuals from different cultures should be taken into consideration by teachers during the learning process (Zhang, 2001). In addition, Borromeo Ferri (2015) explained that there are cultural differences in mathematical thinking style in the study she conducted within the scope of the MaTHSCu project. It has been stated that South Korea and Japan are dominant in the integrated thinking style, whereas Germany is more dominant in visual thinking (Borromeo Ferri, 2015). In the present study, most students were dominant in the integrated thinking style and fewest in the analytical thinking style. In the study of Borromeo Ferri (2015), the mathematical thinking styles of the students were not considered according to gender, so gender comparisons could not be made. However, mathematical thinking styles of students in Turkey can be compared with students from different cultures using this scale. Thus, the influence of culture on mathematical thinking style can be examined in more detail through a gender perspective. Besides, using MTS scale, the reasons the distributions of mathematical thinking styles between boys and girls are different can also be studied. It is also possible to examine male and female students' mathematics achievement according to these mathematical thinking styles. This study shows how LCA can be used to determine students' mathematical thinking styles. This way, it may be easier to identify the mathematical thinking styles of large samples. In addition, the interaction of mathematical thinking styles with different variables can be studied more easily and practically.

Mathematics educators and teachers in Turkey can enrich their knowledge in mathematical thinking styles, and by using this knowledge they can design their courses in a way that students can learn better. However, the information obtained from this study may not directly overlap with other cultures, as styles and styles' effects on success may vary from culture to culture. Therefore, how styles vary in different cultures by such variables as gender would be a valuable topic for further research.

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