Education and Science tedmem

Vol 45 (2020) No 202 417-439

Effects of Number Sense-Based Instruction on Sixth-Grade Students' Self-Efficacy and Performance *

Çiğdem Alkaş Ulusoy [1]

Abstract

In this study, the effects of number sense-based instruction on students' mathematics-related self-efficacy and performance were studied. Self-efficacy relates to both mathematics and number sense, while student performance consists of their number sense, problem solving, general mathematical achievement, and recognition of mathematics in daily life. This was an experimental study where both quantitative research and pretest/posttest control group designs were utilized. The fifteen-week study was conducted with a group of sixth-grade students attending a public school located in the Çankaya district in Ankara Province, Turkey, including thirteen weeks to implement the instructional plan and two weeks for employing the scales. During this process, while students in the test group were introduced to number sense-based training, the control group adhered to the regular syllabus where teachers based most of their instruction on textbooks. The study, beyond its demonstration of number sense, also evaluates number sense development in the context of an instructional plan designed for long-term implementation and contributes to the study of changes in other variables involved in development. Throughout the study, the Number Sense Test was used to identify the number sense of students. The Mathematics Self-Efficacy Perception Scale was used to assess their self-efficacy. The Number Sense Self-Efficacy Scale was used to identify their number sense self-efficacy. The Mathematics in Daily Life Survey was used to assess their recognition of mathematics in daily life. The Problem-Solving Test was used to assess their achievement in solving problems, and end-of-term grading was used to identify their overall mathematical achievement. Data were analyzed with multivariate analysis of variance (MANOVA) tests using SPSS 15.0 (SPSS Inc., Chicago, Il). The results of the analyses revealed that students who received instruction based on number sense experienced significant improvement in their number sense, their recognition of mathematics in daily life, and their problem-solving achievements compared to students who received only regular instruction. Number sense-based instruction did not create any significant changes in mathematics self-efficacy, number sense self-efficacy, or the overall mathematical achievements of the students.

Keywords

Number sense
Number sense-based instruction
Self-efficacy
Mathematics in daily life
Problem solving

Article Info

Received: 12.24.2018
Accepted: 11.20.2019
Online Published: 04.06.2020

DOI: 10.15390/EB.2020.8381

* This article is derived from Çiğdem Alkaş Ulusoy's PhD dissertation entitled "Effects of number sense-based instruction to sixth grade students self efficacies and performances", conducted under the supervision of Mesture Kayhan Altay.

[1] TED University, Faculty of Education, Department of Mathematics and Science Education, Turkey, cigdemalkas@gmail.com

Introduction

Number sense is a relatively new concept, with related studies increasing significantly in recent years. In the book *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM, 2000), features of children who have strong number sense were explained as follows:

> *Children with number sense (1) have a very good understanding of the meaning of numbers, (2) develop multiple relationships between numbers, (3) recognize relative magnitudes of numbers, (4) understand the effects of operations on numbers, and (5) develop points of comparison (benchmarks) for the measurement of objects around them (p. 38).*

Neurologists and psychologists have formed diverse opinions on the origins of number sense. Dehaene (1997) is a neurologist and mathematician who argues that the human brain has a center that perceives numbers. Calculations related to numbers are actuated by stimulation of neuron cells in our cerebral cortex that are associated with numbers. As per this opinion, number sense is a completely biological apparatus related with the brain's structure. Conversely, other opinions oppose Dehaene's view, arguing that number sense cannot be restricted to a biological apparatus. Instead, number sense should be interpreted as knowledge and skill. Mathematics educators usually adopt this view by interpreting number sense as a dynamic feature that can be developed (Yang, 1995). Many mathematicians acknowledge that students' number sense increases with their grade level (Reys et al., 1991; Sowder, 1992). According to Baroody and Coslick (1998), number sense begins with a sense of number magnitude, and the capability of students to develop this sense relies on their meaningful experiences with numbers.

While common elements are found in the arguments of many researchers who study number sense, it is unfortunately difficult to encounter the same definitions in any two studies. Throughout the literature, dissimilar number sense definitions and classifications by numerous researchers are found (Berch, 2005; Case, 1998; Greeno, 1991; Hope, 1989; Howden, 1989; Kayhan Altay & Umay, 2013; McIntosh, Reys, & Reys 1992; Reys et al., 1999; Yang, 1995). In particular, Yang (1995) diverted from other researchers by assessing current definitions and classifications of number sense by their common features and presented a broad framework to approach the concept of number sense. This also constitutes the theoretical basis for this study. Yang (1995) described number sense as an individual's capability and tendency to use numbers and operations flexibly while performing mathematical reasoning and also the ability to develop beneficial and useful strategies in situations requiring mathematics. Furthermore, he organized number sense under a classification comprising six components as follows: (1) *understanding the meanings of numbers*, (2*) decomposition and recomposition of numbers,* (3) *magnitude of numbers,* (4) *comparison,* (5) *understanding the effects of operations on numbers,* and (6) *flexibility in applying the correct number and operation knowledge to situations of calculation*.

Of these components, *understanding meanings of numbers* refers to the ability to understand quantities represented by numbers. In explaining this component, Yang (1995) states that it is developed for individuals who can evaluate the amounts represented by numbers in different contexts. The component *decomposition and recomposition of numbers* refers to the ability to use different representations of numbers flexibly and to select an adequate representation that facilitates calculation. For example, in the operation 240 x 0.25, considering the equation 0.25 = $\frac{25}{100}$ or the operation 24 x 25, unbundling the numbers as 6 x 4 x 25 before recombining them as 6 x 100 are options that may only be selected by a student with advanced number sense. The component *magnitude of numbers* contains the skill of comparing the numbers and grading the numbers. For example, in the operation 534.6 x 0.545 = 291357, a student who does not know where to put the period sign most likely does not have the ability to understand the magnitude of numbers. *Comparison* includes the use of appropriate numbers as benchmark points. For example, when adding the fractions $\frac{8}{9}$ and $\frac{13}{14}$, a student who uses the number 1 as a benchmark may estimate that this sum is slightly lower than 2 because each fraction is slightly lower than 1. *Understanding the effects of operations on numbers* refers to the ability to recognize how the result will differ for a calculation when the value of a number or operation is changed. For example, the students should know that the sum will be less than 100 when they add two numbers of lower value than 50. The last component, *flexibility in applying the number and operation knowledge to situations of calculation,* refers to the skill to select the most efficient and accessible tool of calculation, decide on whether a precise or estimated result will be an appropriate answer when solving a problem, select an adequate strategy, test the relevance of the result, as well as mental calculation and estimation (Yang, 1995, as cited in Alkaş Ulusoy&Şahiner, 2017)

Reys (1994) indicated that the best way to improve students' number sense is to provide them with a process-oriented learning environment where thinking, discovering, interpreting, and meaningful discussions are found. By reviewing previous studies of the development of number sense (Hope & Small, 1994; Gurganus, 2004; Reys, 1994; Sowder, 1992), we can summarize the features of learning environments organized for development of number sense as follows.

First, the class culture or style should be organized in such a way that students are able to think, talk, and freely discuss numbers. Students should be able to express themselves comfortably and talk frequently about numbers and quantities. Collaborative methods like group work and class discussions should be included to ensure that students are aware of each other's operation strategies. It is also important to encourage students to use various strategies in their operations, express the value of estimation when trying to arrive at their answers, and promote additional dialog about the estimation methods they choose.

In order for students to truly understand numbers and multiplicities, a common question is what a number or quantity actually refers to in daily life. The use of materials allows concrete examples of quantities and operations. Different representations of numbers can be used in lessons, and the transformation between these representations is important. All support the effective use of the number line. In addition, numerous activities can develop the sense of numbers, including the following: determine the most appropriate way to organize, sort, and reassemble numbers when dealing with problem solving; explore the relationship between number patterns and numbers; work on realistic math problems using different solutions; and test quantitative results when working with problem situations. It is easy to predict that number sense will be developed with the help of learning environments created with all of these conditions and appropriate number sense activities.

Reviewing the literature, we found that studies on number sense can be grouped under a number of core topics. These include studies on the definition of number sense and identification of its components (Berch, 2005; Case, 1998; Greeno, 1991; Hope, 1989; Howden, 1989; McIntosh et al., 1992; Şengül & Gülbağcı Dede, 2013a; Reys et al., 1999), studies on assessment of number sense and identification of its components and strategies used (Huang & Yang, 2018; Kayhan Altay & Umay, 2013; Tsao, 2005; Yang, 2007; Yang, Reys, & Reys, 2009), studies related to intercultural research of number sense (Aunio, Ee, Lim, Hautamaki, & Van Luit, 2004; Markovits & Pang, 2007; Reys et al., 1999), studies on the development of number sense (Diezman & English, 2001; Chen, Yan, & Xin, 2016; Kaminski, 2002; Markovits & Sowder, 1994; Reys, Kim, & Bay, 1999; Tsao, 2004a; Yaman, 2015; Yang, 2002), and studies that associate number sense with other specific concepts (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Pike & Forrester, 1997; Reys & Yang, 1998; Yang, Li, & Lin, 2008).

This study is oriented toward the development of number sense and observes the changes of selected variables during this process. By reviewing the concepts associated with number sense in the literature, we frequently encountered certain concepts such as estimation, calculation fluency, and mathematical achievement (Jordan et al., 2007; Pike & Forrester, 1997; Reys & Yang, 1998; Yang et al., 2008). In this study of number sense-based instruction, the following were evaluated: self-efficacy toward mathematics and self-efficacy toward number sense under the broader topic of general self-efficacy and its effect on variables of recognition of mathematics in daily life, mathematical achievement, and problem-solving achievements. The concept of the mathematics self-efficacy variable is described by Hackett and Betz (1989, p. 262) as ''a situational or problem-specific assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular task or problem." Self-efficacy related to number sense is derived from the mathematics self-efficacy definitions of Hacket and Betz. It can be defined as the self-confidence of an individual in subjects related to the use of numbers and operations in a flexible manner of mathematical reasoning to develop beneficial and useful strategies in situations involving mathematics.

Veloo (2010) investigated the relationship between number sense and mathematical behavior of secondary school students. As a result of the research, a medium-level relationship was found between "confidence in learning and achieving in mathematics," "confidence in achieving mathematical tasks," the sub-dimensions of the scale, and number sense of the students. Similarly, Tsao (2004b) examined the relationship between number sense and mathematical behaviors of pre-service teachers. In this study, one of the sub-dimensions of mathematical behaviors was confidence in mathematics learning. As a result of the study, a moderate relationship was found between number sense and confidence in mathematics learning. Based on the related literature, the relation between number sense and the self-efficacy concept has been included in the content of this study.

The variable of recognition of mathematics in daily life is studied as one of the variables under the topic of performance. It is defined as an individual's awareness of the role and place of mathematics in daily life (Erturan, 2007). Mousley (2004) listed the connections required for development of mathematical understanding as follows: (1) connection between existing knowledge and new knowledge, (2) connection between different mathematical ideas and representations, and (3) connection between the mathematics in school and mathematics in daily life. Additionally, the variable of recognition of mathematics in daily life was chosen on the pretext that the experiences of individuals play a significant role in the development of number sense, and these experiences will be further strengthened with awareness of mathematics in daily life. Other variables of the study, namely problem-solving achievements and mathematical achievement, were taken in their universal contexts. Problem-solving achievements were determined by student scores on an achievement test developed by the researcher that covered problems related to the subject of numbers, whereas mathematical achievement was assessed by the students' end-of-term scores in the mathematics course.

In NCTM's (2000) Numbers and Operations Standard for students from pre-school to twelfth grade, emphasis for subjects related to numbers is placed on understanding different representations of numbers, connections between numbers and numerical systems, meanings of numbers and their interconnections, and the students' ability to calculate fluently and make sensible estimations. These requirements are components that constitute number sense. In this study, a comprehensive instructional plan for the development of number sense that encompasses all components of number sense was prepared. These plans were implemented with the students for approximately half the school year. During this lengthy period, the students participated in many rich experiences associated with number sense. These opportunities were considered an advantageous factor of the study. Studies conducted related to number sense usually focus on descriptions or assessments. As an experimental study oriented toward development of number sense, this characteristic may be considered another merit of the current study. In previous research, the average number sense of students was found to be quite low (Harç, 2010; Singh, 2009; Yang, 2005; Menon, 2004; Mohamed & Johnny, 2010; Verschaffel, Greer, & DeCorte, 2007). Based on a prediction that this situation arose from the lack of appropriate opportunities for students to develop their number sense, this type of experimental research supporting its development was overdue.

Furthermore, no study was encountered in the literature that evaluates the effects of number sense on students' faith in mathematics and self-efficacy for number sense, mathematics achievement, success in problem solving, and recognition of mathematics in daily life in a holistic manner. It is thought that improvement and intensification in studies that relate to a concept that is identified as having a correlation with a plethora of skills and achievements like mathematics achievement, calculation skills, and estimation skills may contribute to the field of mathematics education. Therefore, answers were sought for the following questions:

When learning numbers and operations, are there any significant differences between sixth-grade students who experience number sense-based instruction and those who don't in the following topics?

1. Number sense
2. Mathematics self-efficacy
3. Number sense self-efficacy
4. Recognition of mathematics in daily life
5. Achievement in problem solving
6. Mathematical achievements.

Method

Students were not randomly assigned to classrooms. Instead, the experimental group was formed from students already found in the classrooms. This paper is therefore an example of a quantitative study with a quasi-experimental design (Fraenkel & Wallen, 2006). That design combined with a pretest-posttest control group was used. The participants were selected from a public school located in Çankaya district of Ankara Province. The school is located in an upper-class socioeconomic environment. As the number of volunteer teachers was restricted and the experimental study was scheduled to be conducted over a long period of time (fifteen total weeks; with thirteen weeks for implementation and two weeks for data collection), the participants were selected from a conveniently accessible school. The classroom of one volunteer teacher was designated randomly as the experimental group (N = 35) and the other was designated as the control group (N = 35).

Data Collection Tools

The following were used as data collection tools: Number Sense Test, Mathematics Self-Efficacy Perception Scale, Number Sense Self-Efficacy Scale, Mathematics in Daily Life Survey, Problem-Solving Test, and end-of-term grading.

Number Sense Test (NST)

The NST was developed by Kayhan Altay and Umay (2013) to identify students' number sense. This test is the only Turkish-language scale used for identifying number sense of primary school students and the only one designed by using factor analysis. Although other current scales were created on the basis of theoretical infrastructures without applying factor analysis, the Altay and Umay test based on factor analysis has become more significant because of its structuring. The scale comprises seventeen open-ended and multiple-choice questions. Consequent to the applied factor analysis, topics were clustered under three components. The researcher named these factors as follows: flexibility in calculation, using benchmark points, and conceptual thinking in fractions. To determine the reliability of the scale, Cronbach-α reliability coefficient was calculated, and the result was expressed as 0.86. A reliability of 0.86 for a seven-topic scale is considered very good (Büyüköztürk, 2007). In this study, the reliability coefficient was calculated as 0.82. For analyzing data associated with the scale, the status of using number sense was considered. Students that used number sense strategies (benchmark use, decomposing and recomposing numbers, estimation, etc.) to explain their solutions to problems were given 1 point, while students who solved problems by calculation in a standard-routine (rule-based) manner were given 0. Accordingly, the highest possible score from the test would be 17 and the lowest 0.

Number Sense Self-Efficacy (NSSE) Scale

The NSSE data collection tool was developed by researchers Alkaş Ulusoy and Şahiner (2017) to determine the number sense self-efficacies of sixth-, seventh-, and eighth-grade students. The scale topics usually provide the student with information based on a specific situation and ask the students to rate their self-efficacy when confronting the situation. For example, "When I am asked to apply the operation 162 + 98, nothing other than writing the numbers one under the other comes to my mind," or "On a number line where only numbers 0 and 100 are marked, I can mark the approximate location of number 78." Students were asked to read these statements and to rate their agreement on a five-point Likert scale. The Cronbach-α reliability coefficient of the test comprised of nineteen topics is 0.82. In this study, it was a satisfactory 0.79.

Mathematics Self-Efficacy Perception (MSEP) Scale

The MSEP was developed by Umay (2001) to assess students' mathematics self-efficacy perceptions. It comprises fourteen topics in three dimensions, including mathematics self-conception, awareness of behavior in mathematics subjects, and transforming mathematics to life skills. The scale uses five-point Likert scales, and each topic is scored from 5 = Always to 1 = Never. As per this scoring system, the lowest possible total score on the scale is 14 and the highest is 70. Umay stated that the Cronbach-α reliability coefficient of the scale is 0.88. The reliability coefficient in this study was identified as 0.80.

Mathematics in Daily Life (MDL) Survey

The MDL survey scale was developed by Erturan (2007) to determine students' ability to recognize the use of mathematics in daily life. It comprises three chapters. In the first chapter, students are asked eight questions associated with daily life that also relate to sixth-grade mathematics subjects. In the second chapter, the students are asked to record the daily activities where they make use of mathematics. In the third and final chapter, students are given ten daily-life situations and asked whether they can use mathematics in these situations. Those who answer "yes" are required to explain how they can use mathematics. Although the researcher worked with seventh-grade students when developing the scale, it is also suitable for sixth-grade students because only the first part of the scale includes mathematical gains. These cases are based on topics covered in the fifth-grade curriculum, such as natural numbers, fractions, environment, and area calculations.

In the MDL scoring system, students who fail to answer eight questions in the first chapter or give wrong answers are given 0 points, students with approximate answers are given 1 point and students that answer correctly are given 2 points. In the first chapter of the survey, the minimum score a student can receive is 0 and the maximum is 16. In the second chapter where students are asked to record their mathematics-related daily activities, those related to mathematics are assigned 1 point while those that are not are assigned 0. In the last chapter where students are asked to explain if and how they use mathematics in ten diverse situations, their explanations are assigned 1 point if they truly involve mathematics and 0 if not.

Problem Solving Test (PST)

The PST was developed to determine student achievement in solving problems related to numbers and operations learning area. Designed as an achievement test, PST includes natural numbers, integers, fractions, decimal fractions, percentages, and ratio and proportion, all of which are sub-topics of the numbers and operations learning area. The scale includes fifteen problems that were found within acceptable values in both difficulty index and discrimination index consequent to the material analysis. The reliability coefficient of the scale is measured at 0.83.

End-of-Term Grading (ETG)

ETG refers to the end-of-term scores for the mathematics course retrieved from student report cards at the end of the term after completion of the experimental study. Each student completed three written exams, one performance assignment, and one classroom performance score. All scores were entered in the school's electronic grading system automatically to create the end-of-term grades. The lowest grade a student can receive is 1 and the highest is 5.

Implementation Process

The first phase of the study was to prepare a plan incorporating all student activities, materials, and evaluation processes in a systematic way within a specified theoretical framework. Upon review of related literature, several approaches were observed regarding the delivery of number sense concepts through various perspectives and different components. Under these circumstances and considering that working with number sense concepts is difficult in itself the framework for this study was restricted to the definition of number sense and its components described by Yang (1995). The reason to choose Yang's definition and components as the framework of this study is that they, albeit under different topics, coincide with the definitions and components of many researchers and form a synthesis of those definitions. This conclusion was reached on the basis of a comprehensive literature review. Following the designation of a suitable framework, the class level at which the study would be implemented was decided. When considering the appropriate level, the following were primary factors: it should be a grade where the students are not concerned with any placement exams; they should not encounter coursework guided by a syllabus that concentrates strictly on operations knowledge and test techniques; and this class should not be their first introduction to subjects where number sense can be incorporated. Given this criteria, the decision was made to implement the study in sixth-grade classes. Following the class level decision, the designation of suitable subject content to infuse number sense concepts was undertaken. Previous number sense studies included subjects related to number systems, such as natural numbers and integers, number magnitude, and representations like fractions, decimal fractions, percentages, and ratios, as well as four operations and story problems regarding those basic operations. In the elementary schools mathematics education program (Ministry of National Education [MoNE], 2013) used at the time this research was conducted, the learning area that best covered all of these subjects was "numbers and operations" learning area. Following this decision, the design stage was initiated to develop a number sense-based instructional program. This phase began by relating objectives in sub-learning areas that constitute the subject content with the components described in Yang (1995). For example, the objective of "*comparing and arranging fractions, and showing them on a number line*" was correlated with the "*understanding the meanings of numbers, number magnitude, and comparisons*" components of number sense. The detailed explanation relevant to this correlation is shown in Table 1.

Table 1. An Example of Objectives: Number Sense Component Relationships

Objective	Number Sense Component	Explanation
Compare fractions, arrange, and show them on the number line.	Understanding the meanings of numbers	Before comparing any two fractions, the student should be able to understand the amount that the fraction represents.
	Number magnitude	Be aware of the size of the fraction when comparing and sorting two or more fractions using flexible strategies instead of denominator equalization for comparison.
	Comparison	Use reference points like 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 when comparing fractions.

Following the objective/number sense correlation for each objective, some of the objectives were rendered individually while some were combined. A total of twenty-five lesson plans oriented toward both objectives along with the development of a related number sense component were created. Lesson plans were reviewed by three experienced teachers and two field specialist scholars who have published previous studies related to number sense. The panel assessed the lesson plans for class level suitability, appropriateness for the study's objectives, and compatibility with specific number sense components. All necessary revisions based on their comments were made to the lesson plans.

A pilot implementation was conducted parallel to the main study. A significant portion of the lesson plans were tested in unrelated sixth-grade classroom that was not involved in the actual study. The researcher attended as an observer when the lessons were taught in the pilot classroom, took notes regarding shortcomings, and used these notes to revise the lesson plans and provide the teacher with pertinent feedback. The school administration provided assistance to ensure that the course schedule was arranged to hold the mathematics lessons of the pilot class before the experimental group and after the control group. Accordingly, the teacher would teach in the conventional method to the control group followed by the pilot study class and ultimately the experimental group each week.

The researcher also attended the first lessons in both the experimental and control groups with the participating teacher. After introduction, she explained that she would participate in the classes as an observer for the purpose of implementing a study throughout the first term. Within a month of the school's starting date, the students were being taught subjects that were not included in the study's scope. During this one-month period, despite some of the covered topics being outside of the study, the researcher still observed the lessons. Therefore, many students considered the researcher as a member of the class from the beginning of the term. These circumstances allowed the study's intervention to be conducted under more comfortable and natural circumstances. Accordingly, it became possible to mitigate any effects that could have been attributed to the presence of an outside observer.

One week prior to delivering the subjects covered by the number sense content, the pretests were completed, after which the subjects within the study's scope commenced. Lesson plans that were supported with activities that aimed to improve number sense were conducted with the experimental group. In the control group, the teacher continued to the lessons guided only by the textbook. In order to eliminate any effect or bias from the teacher, all lessons, assessments, and grading were conducted by the course teacher in both the experimental and control groups. The teacher was first given documentation related to number sense. After reviewing the documentation, related discussions were held. Following this initial information-sharing period, the teacher and the researcher collaborated on a weekly basis to establish the next lesson plan to be taught. Each week, the lesson plan for the upcoming week was presented to the teacher. Also, the most remarkable points, potential questions, and student comments from the previous week were discussed, as were potential difficulties. In this way, the teacher was fully prepared for each lesson.

In the experimental group, the teacher delivered lesson plans created primarily by the researcher. There were four course periods of forty minutes each per week for thirteen weeks. Lesson plans were created by reviewing various books, articles, and theses (Baroody & Coslick, 1998; Boaler, 1994; Bresser & Holtzman, 1999; Carpenter, Lindquist, Matthews, & Silver, 1983; Lesh, Post, & Behr, 1998; Markovits, 1989; NCTM, 2000; Ontario Ministry of Education, 2006a, 2006b, 2006c; Pilmer, 2008; Sertöz, 2002; Van de Walle, Karp, & Bay-Williams, 2010; Yang, 2003, 2006). They were either adopted directly from the source material, adapted from those sources, or newly developed by the researcher. Each lesson began with a "What's the date today?" activity that targeted improvement in the "*understanding meanings of numbers*" component of number sense. In this activity, the number indicated by the current day of the month was selected and students were asked to discuss that number. For example, students' comments on the number 12 were as follows: "twelve is the age of most of us; it also means a dozen; I love this number because it is small but it still has many multipliers: 1, 2, 3, 4, 6, and 12; it comprises 1 ten and 2 ones, and it's an even number." This short activity not only helped facilitate an understanding of number meanings but also fostered student motivation regarding the course and its content. During observations, all students seemed to be very eager to participate in this type of activity. The lesson continued with the distribution of study sheets and materials related to daily life. Students in groups of two or four discussed the study sheets or assignments and took notes about the conclusions they had reached. Following groupwork, their conclusions and created products were discussed with the entire class to ensure that all groups were informed of the work of the other groups. The teacher guided theses class discussions and the corrected conclusions were summarized for the students in mathematical sentences. The lesson was concluded with either a class discussion or a new activity where the conclusions could be applied.

The teacher proceeded on a parallel track in the control group with the same subjects covered in the experimental group, albeit with one major difference. In the control group, the lessons were taught using primarily the textbook published by the Ministry of National Education. During the class, some of the activities in the book were practiced, but the teachers usually included additional techniques, such as direct instruction and question/answer sessions. Classes usually began with a recap of the previous lesson. The teacher then asked the students to write information from the textbook or from her notes, and the classes were concluded by solving exercises given by the teacher. The researcher also participated in the control group as an observer.

Analysis of the Data

Data from this study were analyzed using SPSS 15.0 (SPSS Inc., Chicago, Il). In the study, the descriptive statistics relevant to the data collected from the experimental and control groups were analyzed. Median, standard deviation, skewness, kurtosis, and maximum and minimum values from pretest and posttest scores were identified for both groups. Pretest scores of the two groups were examined on clustered box plots and outliers were also identified. Subsequently, descriptive statistic values associated with difference scores were obtained by subtracting pretest scores from posttest scores of the experimental and control groups. In order to determine whether the difference scores showed normal distribution, skewness and kurtosis coefficients were checked. Following the descriptive statistics that were examined to identify the data, inferential statistics to answer the study's main questions were evaluated. For this study, the MANOVA technique used for variance analysis with multiple dependent variables was utilized. Prior to starting the analysis, a review of the data was required from the perspective of MANOVA's sample size and missing data, normality (univariate and multivariate), outliers, linearity, and the homogeneity of variance-covariance matrices (Stevens, 2002). The MANOVA analyses tested these assumptions.

During collection of the pretest-posttest data and throughout the experiment, there were equal numbers of students in the experimental and control groups (N = 35). However, considering one participant in the experimental group was an inclusive student whose data may have adversely affected the results, the data corresponding to that student were not included in the analysis. All analyses for the experimental group (N = 34) were made with one student less than the control group (N = 35). Furthermore, all data were checked, and no missing data were found in the data set. In order to assess univariate normality, the skewness coefficient was calculated. No scores fell outside the ±1 limits; therefore, they showed no significant deviation from normal distribution. Additionally, in the case of a group size smaller than fifty, the Shapiro-Wilks test can be used to test the scores compliance with normality. As the hypothesis in the analysis was established with the presumption that the scores' distribution might show no significant deviation from the normal distribution, when the calculated p-value is greater than $\alpha = .05$ the scores on this significance level are considered to have no significant deviation from normal distribution (Büyüköztürk, 2007). In this study, all values calculated in relation to pretest-posttest difference scores in the Shapiro-Wilks test were found to be larger than $\alpha = .05$. Accordingly, the univariate normality assumption was satisfied. In order to determine whether the variables showed a multivariate normal distribution, a check of the presence of outliers associated with variables was suggested. Therefore, outliers that satisfied the linearity assumption may also be detected (Büyüköztürk, 2007). For this purpose, the Mahalanobis distances for all dependent variables to be used in the MANOVA analyses were first calculated. Later, the obtained values were compared with the X^2 (p = 0.001, sd = 6) = 22.457 value determined by reviewing the X^2 table. The Mahalonobis value was identified as the multivariate outlier of values larger than 22.457. Examination of the obtained Mahalonobis distances showed no outliers. Linearity between two variables may be reviewed with the help of scatter plots. Both variables show normal distribution, and if there is a linear relationship between two variables, the scatter plot will have an oval shape (Tabachnick & Fidell, 2007). On examination of the scatter plots of all dual relations of dependent variables, the plots were all oval shaped; therefore, there was no condition that would challenge linearity. In order to check whether the homogeneity of variance matrices was satisfied, the Levene test statistics were examined. Since the difference scores of variables (other than problem-solving achievement and number sense variables) were $p > 0.05$, the condition of homogeneity of variance matrices was satisfied. However, the $p > 0.05$ condition could not be satisfied for problem-solving achievement (p = 0.000) and number sense (p = 0.003). In cases when the homogeneity of variance matrices cannot be provided, a logarithmic transformation is suggested (Tabachnick & Fidell, 2007). It was applied to these two variables, and the Levene test was repeated with data obtained from the transformation. New statistics obtained after the test revealed that the hypothesis of homogeneity of variance matrices is satisfied. On the other hand, Box's M test was used to test the hypothesis of homogenity of covariance matrices. In this test, if the p (sig) value is lower than 0.05, the hypothesis is not verified. If the p (sig) value is higher than 0.05, the hypothesis is verified. In this test, as the p (sig) value was $p = 0.052$ and $p > 0.05$, we consider the assumption of homogeneity of variance matrices was satisfied.

After a conclusion was reached that all assumptions were satisfied, statistics from the MANOVA analyses and the analysis of variance (ANOVA) statistics for dependent variables were reviewed to seek answers to the developed hypotheses.

Validity and Ethics

Internal validity assesses whether the effect on the dependent variable is actually caused by the independent variable. In order to ensure validity of the study, any element that may challenge internal validity—such as the features of test subjects, loss of test subjects, space, effect of data collection tools, features and prejudices of practitioners, and the effects of time and maturity—should definitely be controlled (Fraenkel & Wallen, 2006). In this study, as the average pretest scores of the test subjects in the experimental and control groups were very similar, we can argue that the test subjects' features imposed no challenge to this study. During collection of the pretest-posttest data and throughout the experiment, there were equal numbers of students in both the experimental and control groups (N = 35). However, as mentioned, one of the participants in the experimental group was an inclusive student whose data may have adversely affected the results; therefore, the data of that student were excluded in the analyses. This left the experimental group (N = 34) with one less student than the control group (N = 35). Although the data of this participant from the experimental group were lost, as the total number of students in each group were still very similar, this loss was not seen as an impediment that would challenge internal validity. The interventions for both the experimental and control groups were conducted in the regular classroom of each group; therefore, this eliminated any potential environmental factor that might have imposed any concerns for the students.

Many data collection tools were used in the study, and their application was time consuming. A time period of one week was estimated for data collection. In the experimental and control groups, the same data collection tools were used on the same days. Since the lesson plans used in the study were scheduled over a lengthy period of thirteen weeks, the students' learning of the data collection tools and any effects on the results of the posttest were unlikely. When collecting data with the data collection tools, no events that may have affected the students' answers occurred. Furthermore, it was already established that the data collection tools were all valid and reliable. Therefore, we could safely state that the data collection tools posed no threats to this study.

The researcher participated as an observer in both the experimental and control groups from the beginning of the school term. Therefore, the students identified the researcher as a member of the class. There were no prior connections between the researcher and any of the participants. Accordingly, the researcher had no prejudice toward any test subjects that may have threatened the internal validity of the study. Students in both groups were the same age. As the study included various stages of data collection and implementation spanning fifteen weeks, their maturity posed no threats to the study. External validity relates to the generalization of results obtained from a study (Fraenkel & Wallen, 2006). In this study, variables used to enhance external validity, implementation, working environment, and conditions were explained with as much detail as possible. This study was assessed and approved by the Hacettepe University Ethics Committee based on its compliance with all ethical standards. Furthermore, mandatory permissions were obtained from the National Ministry of Education and the supervisory authorities of the school where the study was conducted. All teachers and students who participated in the study were duly informed about the intentions of the study. Furthermore, during data collection, the students were told that the data they supplied would be used exclusively by the researcher for scientific purposes, and their names and personal data would not be shared with any individual or entity in any manner. Students who did not want their data used for any reason whatsoever could have their data excluded from the study with no explanation necessary.

Results

The average, standard deviation, skewness, kurtosis, and maximum and minimum values from the pretest and posttest scores of the dependent variables obtained from the experimental and control groups are summarized in Tables 2 and 3. As observed in Table 2, the average pretest scores of the dependent variables obtained from the two groups prior to implementation were quite similar. The average value with the largest difference was the mathematics self-efficacy score (51.59 in the experimental group and 55.17 in the control group, respectively).

Table 2. Pretest Descriptive Statistics of the Experimental and Control Groups

Groups	*Variables*	*N*	*Average*	*Std. Deviation*	*Min.*	*Max.*	*Skewness*	*Kurtosis*	*Max. Score*
Exp. Group	Pre-NS	34	2.38	1.43	0	5	.18	-.74	17
	Pre-MSEP	34	51.59	8.53	31	64	-.39	-.60	70
	Pre-NSSE	34	70.44	10.39	43	91	-.64	.25	95
	Pre-MDL	34	15.03	8.53	3	33	.70	-.56	46
	Pre-PSA	34	2.44	1.46	0	5	.09	-.88	15
Cont. Group	Pre-NS	35	2.43	1.33	0	5	.21	-.76	17
	Pre-MSEP	35	55.17	8.59	33	70	-.75	.44	70
	Pre-NSSE	35	71.37	9.74	49	85	-.72	-.21	95
	Pre-MDL	35	15.11	8.17	5	32	.72	-.54	46
	Pre-PSA	35	2.22	1.33	0	5	.26	-.45	15

Table 3 represents average, standard deviation, skewness, kurtosis, and maximum and minimum values from posttest scores obtained from the experimental group and control groups. Compared to Table 2, there were increases in the average scores of all independent variables in the experimental group as follows: number sense increased from 2.38 to 10.62, mathematics self-efficacy increased from 51.59 to 10.53, number sense self-efficacy increased from 70.44 to 82.50, recognition of mathematics in daily life increased from 15.03 to 18.47, and problem-solving achievement increased from 2.44 to 7.50. In the control group, increases were observed in average scores for number sense, number sense self-efficacy, and problem-solving achievement, while there were decreases in the average scores for mathematics self-efficacy and recognition of mathematics in daily life.

Table 3. Pretest Descriptive Statistics of the Experimental and Control Groups

Groups	*Variables*	*N*	*Average*	*Std. Deviation*	*Min.*	*Max.*	*Skewness*	*Kurtosis*	*Max. Score*
Exp. Group	Post-NS	34	10.62	3.52	2	17	-.02	-.19	17
	Post-MSEP	34	60.53	6.98	44	70	-.72	-.07	70
	Post-NSSE	34	82.50	10.81	61	95	-.57	-.74	95
	Post-MDL	34	18.47	9.13	5	36	.47	-.95	46
	Post-PSA	34	7.50	3.76	1	15	.09	-.53	15
	MA	34	3.91	.96	2	5	-.46	-.73	5
Cont. Group	Post-NS	35	4.58	1.86	1	8	.46	-.18	17
	Post-MSEP	35	54.69	6.98	35	66	-.89	.90	70
	Post-NSSE	35	81.71	8.16	60	95	-.61	.18	95
	Post-MDL	35	13.74	6.52	3	28	.64	-.32	46
	Post-PSA	35	5.91	1.59	4	10	.79	.07	15
	MA	35	3.53	1.08	2	5	.00	-1,23	5

The maintenance of the skewness coefficient an indicator of normal distribution of continuous variables—within the ±1 limits (Büyüköztürk, 2007) can be checked in Tables 2 and 3. All three skewness coefficients in the tables were within the ±1 limits; therefore, no significant deviations from normal distribution of the scores were found.

Table 4 presents the relevant descriptive statistical values associated with score differences created by subtracting the experimental and control group pretest scores from their respective posttest results. When the skewness and kurtosis values were examined, the skewness coefficients remained within the ±1 limits; therefore, the scores showed no significant deviation from normal distribution.

Table 4. Descriptive Statistical Values Associated with Difference Scores in the Experimental and Control Groups

Groups	*Variables*	*N*	*Average*	*Std. Deviation*	*Min.*	*Max.*	*Skewness*	*Kurtosis*
Exp. Group	D-NS	34	9.09	2.70	2	15	-.17	.66
	D-MSEP	34	8.69	3.49	2	17	.36	.05
	D-NSSE	34	13.88	4.98	6	23	.08	-.98
	D-MDL	34	3.44	2.85	-3	9	.00	-.51
	D-PSA	34	5.32	3.01	1	11	.30	-.95
Cont. Group	D-NS	35	1.97	1.36	-1	5	.54	-.18
	D-MSEP	35	-.49	6.32	-11	15	.45	-.36
	D-NSSE	35	5.00	9.29	-13	32	.51	.84
	D-MDL	35	-1.37	3.03	-7	4	-.16	-.85
	D-PSA	35	3.31	1.56	0	6	-.23	-.74

Inferential Statistics

Following review of the MANOVA hypotheses (detailed in the titles of the analyses of the data), a MANOVA analysis was applied with data associated with dependent variables D-MDL, MA, D-MSEP, D-NSSE, L-PSA, and L-NS. Statistics of the analysis are shown in Table 5.

Table 5. Statistics of the MANOVA Analyses

Source of Variance	*Wilks' λ*	*F Value*	*Hypothesis df*	*Error df*	*p Value*	*Eta Square*
GROUP	.217	37.335	6.000	62.000	.000	.783

When assessing the MANOVA results, the significance (p) value of the Wilks' Lambda test was checked. If the p-value is lower than 0.05, it is assumed that there is a significant difference between at least one of the dependent variables between at least two groups of the independent variable. In order to interpret the significance value with better accuracy, a Bonferroni correction is advised and can be calculated by dividing the alpha significance value to the number of dependent variables (Tabachnick & Fidell, 2007). Accordingly, the significance (p) value of the Wilks' Lambda test would be interpreted by comparing to $0.05 / 6 = 0.008$ value. When Table 5 is examined, for the independent variable GRUP, $F(6.62) = 37.33$ and $p = 0.00$. This indicates that there was a significance difference in at least one of the dependent variable values associated with the experimental and control groups. On the other hand, the eta square value identified the variance ratio of the effects of experiencing number sense-based instruction as 78%. In order to identify the dependent variable that caused this difference after the experiment, Table 6 was reviewed. This table shows statistics related to the variations of the dependent variables.

Table 6. ANOVA Table of the Dependent Variables

Dependent Variable	*sd*	*F*	*p*	*Eta Square*
L-NS	1	161.75	.000	.707
D-MSEP	1	.040	.842	.001
D-NSSE	1	2.27	.136	.033
D-MDL	1	46.09	.000	.408
L-PSA	1	8.16	.006	.109
MA	1	2.26	.137	.033

L-NS: Logaritmik Transformation Number Sense
L-PSA: Logaritmik Transformation Problem-Solving Achievement

Table 6 reveals significant differentiation in number sense variables (F[1.67] = 161.75; p = 0.000), problem-solving achievement (F[1.67] = 8.16, p = 0.006), and the recognition of mathematics in the daily lives of students. By comparing the data in Tables 6 and 7, this differentiation favors the experimental group.

Table 7. Controlled Average and Standard Deviation Values

Dependent Variable	*Group*	*Average*	*Standart Deviation*
L-NS	Exp.	1.030	.026
	Cont.	.569	.025
D-MSEP	Exp.	4.000	1.217
	Cont.	4.343	1.199
D-NSSE	Exp.	7.794	1.474
	Cont.	10.914	1.453
D-MDL	Exp.	3.441	.505
	Cont.	1.371	.498
L-PSA	Exp.	.746	.036
	Cont.	.601	.036
MA	Exp.	3.912	.175
	Cont.	3.543	.172

Since the $p < 0.05$ condition could not be satisfied, no significant differences were observed in the following variables: mathematics self-efficacy (F[1.67] = 0.040, p = 0.842); number sense self-efficacy (F[1.67] = 2.27, p = 0.136); and end-of-term mathematics scores (F[1.67] = 2.26, p = 0.137).

Based on these data, there were significant differences between number sense, problem-solving achievements, and recognition of mathematics in daily life. There were no significant differences between mathematics self-efficacy, self-efficacy toward number sense, and mathematics end-of-term grades.

Discussion and Conclusion

Results of the analyses confirm that number sense-based instruction created a statistically significant differentiation in favor of the experimental group for the students' number sense, mathematics self-efficacy, number sense self-efficacy, recognition of mathematics in daily life, problem-solving achievement, and mathematics achievement compared to students who did not receive this instruction. The dependent variables causing this differentiation were checked and identified as number sense, recognition of mathematics in daily life, and problem-solving skills. Number sense-based instruction caused no significant differences on the other dependent variables of the study, namely, mathematics self-efficacy, number sense self-efficacy, and mathematics achievement. Independent results obtained from the study were presented for each of the dependent variables.

One priority of this experiment was to determine whether number sense-based instruction could significantly improve the number sense of students who received this instruction compared to those who did not. According to the results of the analyses, a significant difference was found in favor of the experimental group that received the number sense instruction. This was the expected outcome based on existing literature. Yang (1995) asserted that number sense is not static; on the contrary, it can be improved. Many previous studies have found that when adequate materials and compliant technological equipment are provided, number sense can be improved (Griffin, 2004; Kaminski, 2002; Markovits & Sowder, 1994; O'nan, 2003; Whitacre & Nickerson, 2006; Yang, 2002; Yang & Tsai, 2010). Yang, Hsu, and Huang (2004) posited in their related studies that the encouragement of students to discover numbers and fractions while thinking and discussing these subjects openly has a significantly positive effect on number sense development. Parallel to this opinion, during the experiment, all efforts were taken to provide students in the experimental group with the opportunity to comfortably express their ways of thinking on numbers during both group and classroom discussions.

One important factor in the development of number sense of the students in the experimental group is that they made more significant progress—particularly in estimation and mental calculation skills—under the following components: "flexibility in applying their number and operation knowledge to calculation" and "comparison" (using benchmarks).

In this study, students were given the opportunity to experience as much as possible about these skills. Related literature also asserts that the main reason for the students' lack of efficacy in estimation skills is insufficient experience (Sowder & Wheeler, 1989; Tsao & Pan, 2010) and the lack of motivation to use their estimation skills to test the significance of their results (Bestgen, Reys, Rybolt, & Wyatt, 1980). At the beginning of the experiment when the experimental group students were asked to make mental calculations, many closed their eyes and used their minds like a notebook by trying to animate the algorithms they had previously applied in their notebooks. Later in the experiment, students had learned to internalize mental computation using different strategies.

Baroody and Coslick (1998) stated that human number sense development is related to meaningful experiences with numbers. As much as possible, specific subjects were emphasized in the experimental group, including the following: numbers, types of numbers, quantities corresponding to numbers, number significance, and situations where numbers are used in daily life. During these activities, improvements in the number sense component "understanding the meanings of numbers" were targeted. Students were asked to identify the numbers they use in their daily lives. Therefore, the students were soon able to associate the numbers they encounter in mathematics textbooks with numbers they encounter in daily life. These types of exercises are thought to be effective in the development of number sense.

Many students are unable to associate the subjects they learn at school with the mathematics they are required to use in daily life. This situation can be explained by the lack of conceptual teaching of mathematical notions beginning at lower levels, and accordingly, the failure of students to adapt classroom concepts to daily life (Erturan, 2007). Numbers and operations are in most cases the mathematical concepts that students use most frequently in their daily lives. However, when these subjects are not learned conceptually and strict adherence to rules prevents flexible exploratory thinking, it is more difficult for students to transfer and efficiently use these concepts in their daily lives.

Whenever possible, activities based on situations that students encounter in their daily lives were taken as the starting point for this study. These included school, shopping, trips, theaters, and fairs where students generally enjoy passing their time. During these activities, students were expected to interpret the numbers and operations they use daily and utilize them flexibly in several ways. For many activities, subjects commonly required in daily life were emphasized, including numbers, operations, the effects of operations on numbers, interrelations of operations, mental calculations, quantity estimates, and estimates of operational results. At the end of the study, significant differences were observed between students who received number sense-based instruction and those who did not in terms of their recognition of mathematics in daily life.

In the literature, several studies concluded that connecting daily use of mathematics with classroom subjects contributes to improvements in number sense. Many researchers made theoretical studies on the influential role played by an individual's experiences in the development of number sense and argued that these opportunities to learn are often best experienced during daily life (Anghileri, 2000; Martinie, & Coates, 2007; Treffers, 1991). In this study, a conclusion was reached that the implementation of number sense activities contributed to the students' recognition of mathematics in their daily lives. Students became more comfortable associating their knowledge of numbers and operations—the mathematical concepts used most often by most students—with practical tasks. This may have contributed to the students' enhanced recognition of mathematics in their daily lives.

By the end of the study, the number sense-based instruction had created a significant difference in problem-solving achievements between students who received the instruction and those who did not. Considering the relationship between number sense combined with certain strategies that students need while solving problems and certain rules they use while rendering these strategies, this result is highly significant. For example, among the number sense components used, those such as "decomposing and recomposing numbers," "number magnitude," and "understanding the effects of operations on numbers" all provided confidence in operations and were also effective when testing the significance and accuracy of mathematical operations. Additionally, number sense components that enabled flexible thinking for solving problems may have improved the students' problem-solving achievements. Examples are "estimation" and "using a benchmark point." Furthermore, another number sense component, "flexibility in applying number and operation knowledge to situations of calculation," encompasses testing the significance of obtained results. This may have allowed the students to test the significance of their results and thereby enhance their problem-solving achievement.

Conclusions reached in this study are aligned with related literature. For example, intuition used during the problem-solving process, especially when used by younger students, was emphasized (Carpenter, 1986). Also, studies have pointed to the strong correlation between number sense and problem-solving achievement (Işık & Kar, 2011; Louange & Bana, 2010).

The number sense-based instruction process created no significant differences in mathematics self-efficacy of students who did or did not participate in the process. In the experimental group, the average mathematics self-efficacy score prior to the experiment was 51.59 and had risen to 60.63 by the conclusion of the experiment. In the control group, the average mathematics self-efficacy score prior to the experiment was 55.17 and actually dropped to 54.69 by the end of the study period. Considering that the highest possible score from the scale was 70, it may be argued that the students in both the experimental and control groups had self-efficacy levels that would generally be considered high even before the experiment. Similarly, Şengül and Gülbağcı Dede (2013b) studied the relationship of eighth-grade students' mathematics self-efficacy with their number sense and observed that the students' mathematics self-efficacy was high. Under these circumstances, trying to increase a self-efficacy score that is already high is very challenging. Furthermore, considering that change in belief-based perceptions such as attitude and self-efficacy span longer time periods, it may be argued that the implementation period was not sufficiently long to notably affect self-efficacy.

Bandura (1997) asserted that individuals with previous successful experiences will have higher self-efficacy. Students in the experimental group were able to express their opinions comfortably when freed from strict adherence to rules. They were more flexible and spoke without fear of giving wrong answers. Accordingly, they had gone through positive personal experiences, a major source of self-efficacy, and felt more personally successful. In the long run, sustaining this type of learning environment will foster positive effects on the students' mathematics self-efficacy.

The number sense-based instruction process created no significant differences between the number sense self-efficacy of the students who experienced the instruction process or not**.** It is widely acknowledged that changing an individual's belief toward any subject takes considerable time. Individuals are first required to gain the knowledge and experience related to the subject, overcome their prejudices toward the subject, develop new beliefs, and then embrace those beliefs. Number sense self-efficacy, like any self-efficacy, is a belief-based concept that may require considerable time to see any significant change. Furthermore, as most students have far more experience in education systems where results have been reached by adherence to rules that are valued higher than new alternative methods to interpret numbers, it is even harder to modify their beliefs. Although certain behaviors that reflected changes in student opinions regarding their numbers self-efficacy were observed during the in-class sessions, results show that these evolving opinions could not be endogenized and therefore were not reflected in the statistical outcomes.

The number sense-based instruction process created no significant differences between mathematics achievements of the students who did or did not experience this instruction. However, the literature includes studies that show a significant relationship between number sense and mathematics achievement (Harç, 2010; Jordan et al., 2007; Jordan, Glutting, & Ramineni, 2010; Mohamed & Johnny, 2010; Yang et al., 2008). Results of this study do not coincide with results reached by those previous studies.

Number sense is interpreted as a skill rather than a fund of knowledge. Reflecting this skill in behaviors is quite difficult and time consuming. Therefore, in this study, number sense skills acquired in a matter of weeks could not be adequately reflected in exam papers. Furthermore, report card scores that represent the students' overall mathematical achievements do not exclusively cover subjects related to the numbers and operations upon which this research was conducted. Report card scores comprise exam results that cover all subjects taught in the mathematics course as well as project assignments, class participation, and conduct marks. Given that the concept of number sense is novel for most students, it may be argued that they could still not transfer their skills related to this concept to other subjects included in other exams. The design and scope of this study was restricted to the learning area specifically related to numbers and operations. One can reasonably suggest that associating number sense activities and practices in the experimental group in other learning areas—including geometry, measurement, or algebra—and utilizing them over a longer period would enhance the overall mathematical achievements of students.

Suggestions

This study found that the number sense of students can be significantly improved by an efficient education system oriented toward its development. The points required for this development can be explained by certain components. First, the instructional program should be designed specifically to enable development of number sense. Even though certain skills like mental calculation, estimation, and various components of number sense are found in many instructional programs, inclusion of number sense with its entire array of components would be more beneficial. Naturally, the inclusion of number sense alone is insufficient. Teachers should also be informed and trained on the concept of number sense and the different approaches it entails. Most teachers who are already familiar with number sense to a certain extent may say that they introduced estimation and mental calculation under their programs and that these contributed to the development of number sense. However, what is expected from teachers in number sense development is not to display it whenever requested but to embrace the concept and apply it in all situations they believe it to be necessary. Accordingly, teachers should first create classroom cultures that enable the development of number sense. In an appropriate classroom environment, students opt for their own solutions. They should not be subjected to algorithms in connection with numbers and operations. They should be able to make use of components—such as estimation, benchmarks, and flexible calculation skills—without resorting to pen and paper unless absolutely necessary. Students should be allowed to discover the effects of operations on numbers; they should be interested in number sense activities that encompass potential situations they will experience in their daily lives; and they should be able to test the results of their operations or problems. The concept of number sense is also very significant for teacher candidates who have yet to commence their professional careers. In essence, teacher candidates are the teachers of the future. They should be introduced to number sense concepts during their undergraduate studies so that these methods become natural to them. Number sense should be integrated into the contents of methodology courses and offered at least as an elective course in all tertiary programs.

In Turkey, there are insufficient resources to enable teachers, teacher candidates, or even parents and students to inform themselves about number sense. New resources are needed that include information on number sense, various number sense activities and games, and guidelines for further number sense development. The work of field specialists can create these types of resources to raise awareness of number sense and contribute to its development. Furthermore, a comprehensive review and assessment of current mathematics textbooks in the context of number sense is also suggested for researchers. Contributions to revise these books can be provided and the criteria to support the development of number sense for future materials can be established.

Number sense is a skill for which development should start at a very early age. In many countries, studies start in either pre-school or primary school (Baroody, Eiland, & Thompson, 2009; Jordan et al., 2010; Locuniak & Jordan, 2008). Studies designed to develop number sense may also be expanded to various class levels in Turkey, in particular, early childhood and primary school levels where interpretation of numbers begins in many countries.

Pen-and-paper tests administered to students to determine their number sense yield results that are an assessment-based representation of their number sense. However, when number sense is researched in the form of a qualitative study, more details can be obtained. For example, we can find whether students actually use their number sense or not, and if they do, how they use it and how they use particular components. Also, the number sense of students can be reviewed in further detail through follow-up interviews.

This study is limited to the learning area of numbers and operations in the mathematics instructional program. Similar studies where number sense activities are designed and implemented in fields like algebra, measurement, or geometry can be prepared, and potential contributions of number sense activities in these fields should be studied.

References

Alkaş Ulusoy, Ç., & Şahiner, Y. (2017). Sayı duyusuna yönelik özyeterlik ölçeğinin geliştirilmesi. *Kastamonu Eğitim Dergisi, 25*(1), 17-32.

Anghileri, J. (2000). *Teaching number sense.* London: Cromwell Press.

Aunio, P., Ee, J., Lim, S. E. A., Hautamaki, J., & Van Luit, J. E. H. (2004). Young children's number sense in Finland, Hong Kong and Singapore. *International Journal of Early Years Education, 12*(3), 195-216.

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: W. H. Freeman & Company.

Baroody, A. J., & Coslick, R. T. (1998). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction.* Mahwah, N.J: Lawrence Erlbaum Associates.

Baroody; A. J., Eiland, M. & Thompson, B. (2009) Fostering at-risk preschoolers' number sense, *Early Education and Development*, *20*(1), 80-128.

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*(4), 333-339.

Bestgen, B. J., Reys, R. E., Rybolt, J. F., & Wyatt, J. W. (1980). Effectiveness of systematic instruction on attitudes and computational estimation skills of preservice elementary teachers. *Journal for Research in Mathematics Education, 11*(2), 124-136.

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female in underachievement in relation. *British Educational Research Journal, 20*(5), 551-564.

Bresser, R., & Holtzman, C. (1999). *Developing number sense, grades 3-6.* Sausalito, CA: Math Solutions.

Büyüköztürk, Ş. (2007). *Sosyal bilimler için veri analizi el kitabı. İstatistik, araştırma deseni, SPSS uygulamaları ve yorum.* Ankara: Pegem A Yayıncılık.

Carpenter, T. P. (1986). Conceptual knowledge as a foundation for procedural knowledge: Implications from research on the initial learning of arithmetic. In J. Hiebert (Ed.), *Conceptual procedural knowledge: The case of mathematics* (pp. 113-132). Hillsdale, NJ: Erlbaum.

Carpenter, T. P., Lindquist, M. M., Matthews, W., & Silver, E. A. (1983). Results of the third NAEP mathematics assessment: Secondary school. *Mathematics Teacher, 76*(9), 652-659.

Case, R. (1998, April). *A psychological model of number sense and its development*. Paper presented at the Annual meeting of the American Educational Research Association, San Diego, CA.

Chen, F., Yan, Y., & Xin, T. (2016). Developing a learning progression for number sense based on the rule space model in China. *Educational Psychology, 37*(2), 128-144. doi:10.1080/01443410.2016.1239817

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Diezmann, C. M., & English, L. D. (2001) Developing young children's multi-digit number sense. *Roeper Review, 24*(1), 11-13.

Erturan, D. (2007). *Yedinci sınıf öğrencilerinin sınıf içindeki matematik başarıları ile günlük hayatta matematiği fark edebilmeleri arasındaki ilişki* (Unpublished master's thesis). Hacettepe University, Ankara.

Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education*. Boston: McGraw-Hill.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain source. *Journal for Research in Mathematics Education, 22*(3), 170-218.

Griffin, S. (2004). Building number sense with number worlds: A mathematics program for young children. *Early Childhood Research Quarterly, 19*(1), 173-180.

Gurganus, S. (2004). Promote number sense. Intervention *In School and Clinic, 40*(1), 55-58.

Hackett, G., & Betz, N. E. (1989). An exploration of the mathematics self-efficacy/ mathematics performance correspondance. *Journal for Research in Mathematics Education, 20*(3), 261-273.

Harç, S. (2010). *6. sınıf öğrencilerinin sayı duygusu kavramı açısından mevcut durumlarının analizi* (Unpublished master's thesis). Marmara University, İstanbul.

Hope, J. (1989). Promoting number sense in school. *Arithmetic Teacher, 36*(6), 12-16.

Hope, J., & Small, M. (1994). *Number sense in interactions: Program information.* Toronto, Ontario: Ginn Publishing Canada Inc.

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher, 36*(6), 6-11.

Huang, J. W., & Yang, D. C. (2018). Methods used by fourth graders when responding to number sense-related questions. *American Journal of Education and Learning, 3*(1), 1-13.

Işık, C., & Kar, T. (2011). İlköğretim 6, 7 ve 8. sınıf öğrencilerinin sayı algılama ve rutin olmayan problem çözme becerilerinin incelenmesi. *Ahi Evran Üniversitesi Eğitim Fakültesi Dergisi, 12*(1), 57-72.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 20*(2), 82-88.

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting firstgrade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice, 22*(1), 36-46.

Kaminski, E. (2002). Promoting mathematical understanding: Number sense in action. *Mathematics Education Research Journal, 14*(2), 133-149.

Kayhan Altay, M., & Umay, A. (2013). The development of number sense scale towards middle grade students. *Education and Science*, *38*(167), 241-255.

Lesh, R., Post, T., & Behr, M. (1998). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93-118). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.

Locuniak, M. N. & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities, 41*(5), 451-459.

Louange, J., & Bana, J. (2010). The relationship between the number sense and problem solving abilities of year 7 students. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: 33rd annual conference of the mathematics education research group of Australasia proceedings book* (pp. 360-366). Fremantle, Australia: Mathematics Education Research Group of Australasia.

Markovits, Z., & Pang, J. (2007). *The ability of sixth grade students in Korea and Israel to cope with number sense tasks.* In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *31st conference of the international group for the psychology of mathematics education (PME 31) proceedings book* (pp. 241-248). Seoul, South Korea.

Markovits, Z., & Sowder, J. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25*(1), 4-29.

Martinie, S., & Coates, G. D. (2007). A push for number sense makes good sense. *Mathematics Teaching in the Middle School, 13*(2), 88-91.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12*(3), 2-9.

Menon, R. (2004). Elementary school children's number sense. *International Journal for Mathematics Teaching and Learning.* Retrieved from http://www.cimt.plymouth.ac.uk/journal/ramamenon.pdf

Ministry of National Education. (2013). *Ortaokul matematik dersi (5, 6, 7 ve 8. sınıflar) öğretim programı*. Ankara: Milli Eğitim Basımevi.

Mohamed, M., & Johnny, J. (2010). Investigating number sense among students. *Procedia Social and Behavioral Sciences, 8*, 317-324.

Mousley, J. (2004). *An aspect of mathematical understanding: The notion of "connected knowing"*. In M. J. Hoines & A. B. Fuglestad (Eds.), *28th conference of the ınternational group for the psychology of mathematics education (PME 28) proceedings book* (pp. 377-384). Bergen, Norway.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM Publications.

O'nan, M. (2003). *Daily number talks and the development of computational strategies in fourth graders* (Unpublished master's thesis). Johnson University, USA.

Ontario Ministry of Education. (2006a). *Number sense and numeration, grades 4-6, multiplication,* Retrieved from http://eworkshop.on.ca/edu/resources/guides/NSN_vol_3_Multiplication.pdf

Ontario Ministry of Education. (2006b). *Number sense and numeration, grades 4-6, fractions.* Retrieved from http://eworkshop.on.ca/edu/resources/guides/NSN_vol_5_Fractions.pdf

Ontario Ministry of Education. (2006c). *Number sense and numeration, grades 4-6, decimal numbers.* Retrieved from http://eworkshop.on.ca/edu/resources/guides/NSN_vol_6_Decimal_Numbers.pdf

Pike, C. D., & Forrester, M. A. (1997). The influence of number-sense on children's ability to estimate measures. *Educational Psychology, 17*(4), 483-500.

Pilmer, C. D. (2008). *Number sense*. Canada: NSSAL.

Reys, B. J. (1994). Promoting number sense in middle grades. *Teaching Mathematics in the Middle School, 1*(2), 114-120.

Reys, B. J., Barger, R., Dougherty, B., Lemdke, L., Parnas, A., Sturdevant, R., … Weber, M. (1991). *Developing number sense in the middle grades*. Reston, VA: National Council of Teachers of Mathematics.

Reys, B. J., Kim, O. K., & Bay, J. M. (1999). Establishing fraction benchmarks. *Mathematics Teaching in the Middle School, 4*(8), 530-532.

Reys, R., Reys, B., McIntosh, A., Emanuelsson, G., Johansson, B., & Yang, D. C. (1999). Assessing number sense of students in Australia, Sweeden, Taiwan, and the United States. *School Science and Mathematics, 99*(2), 61-70.

Reys, R. E., & Yang, D. C. (1998). Relationship between computational performance and number sense among sixth- and eighth- grade students in Taiwan. *Journal for Research in Mathematics Education, 29*(2), 225-237.

Sertöz, S. (2002). *Matematiğin aydınlık dünyası*. Ankara: TÜBİTAK Yayınları.

Singh, P. (2009). An assessment of number sense among secondary school students. *International Journal for Mathematics Teaching and Learning.* Retrieved from http://www.cimt.plymouth.ac.uk/journal/singh.pdf

Sowder, J., & Wheeler, M. (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education, 20*(2), 130-146.

Sowder, J. T. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York: Macmillan.

Stevens, J. (2002). *Applied multivariate statistics for the social sciences.* USA: Lawrence Erlbaum Associates Publishers.

Şengül, S., & Gülbağcı Dede, H. (2013a). An investigation of classification of number sense components. *The Journal of Academic Social Science Studies, 6*(8), 645-645.

Şengül, S., & Gülbağcı Dede, H. (2013b). 7. ve 8. sınıf öğrencilerinin sayı hissi ile matematik öz yeterlikleri arasındaki ilişkinin incelenmesi. *The Journal of Academic Social Science Studies, 6*(4), 1049-1060.

Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics*. USA: Pearson.

Treffers, A. (1991). Meeting innumeracy at primary school. *Educational Studies in Mathematics, 26*(1), 333-352.

Tsao, Y. L. (2004a). Effects of a problem-solving-based mathematics course on number sense of preservice teachers. *Journal Of College Teaching And Learning, 1*(2), 33-50.

Tsao, Y. L. (2004b). Exploring the connections among number sense, mental computation performance, and the written computation performance of elementary preservice school teachers. *Journal of College Teaching & Learning, 1*(12), 71-90.

Tsao, Y. L. (2005). The number sense of preservice elementary school teachers. *College Student Journal, 39*(4), 647-679.

Tsao, Y. L., & Pan, T. R. (2010). The study of the computational estimation performance and computational estimation strategy. *New Wave-Educational Research & Development, 13*(1), 12-42.

Umay, A. (2001). İlköğretim matematik öğretmenliği programının matematiğe karşı özyeterlik algısına etkisi. *Journal of Qafqaz University, 8*(1), 1-8.

Van de Walle, J. A., Karp, K., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally.* Boston: Allyn & Bacon.

Veloo, P. K. (2010). *The development of number sense and mental computation proficiencies: An intervention study with secondary one students in Brunei Darussalam* (Unpublished doctoral dissertation). University of Otago, Dunedin, New Zealand.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557- 628). Charlotte, NC: Information Age Publishing.

Whitacre, I., & Nickerson, S. D. (2006). Pedagogy that makes (number) sense: A classroom teaching experiment around mental math. *PME-NA proceedings, 2*, 736-743.

Yaman, H. (2015). The mathematics education I and II courses' effect on teacher candidates' development of number sense. *Educational Sciences: Theory and Practice, 15*(4), 1119-1135.

Yang, D. C. (1995). *Number sense performance and strategies possessed by sixth and eighth grade students in Taiwan* (Unpublished doctoral dissertation). University of Missouri, Columbia.

Yang, D. C. (2002). Teaching and learning number sense: One successful processoriented activity with sixth grade students in Taiwan. *School Science and Mathematics, 102*(4), 152-157.

Yang, D. C. (2003). Developing number sense through realistic settings. *APMC, 8*(3), 12-17.

Yang, D. C. (2005). Developing number sense through mathematical diary writing. *APMC*, *10*(4), 9-14.

Yang. D. C. (2006). Developing number sense through real-life situations in school. *Teaching Children Mathematics, 13*(2), 104-110.

Yang, D. C. (2007). Investigating the strategies used by pre-service teachers in Taiwan when responding to number sense questions. *School Science and Mathematics, 107*(7), 293-301.

Yang, D. C., & Tsai, Y. F. (2010). Promoting sixth graders' number sense and learning attitudes via technology-based environment. *Educational Technology & Society, 13*(4), 112-125.

Yang, D. C., Hsu, C. J., & Huang, M. C. (2004). A study of teaching and learning number sense for sixth grade students in Taiwan. *International Journal of Science and Mathematics Education, 2*, 407-430.

Yang, D. C., Li, M. N., & Lin, C. I. (2008). A Study of the performance of 5th graders in number sense and its relationship to achievement in mathematics. *International Journal of Science and Mathematics Education, 6*(4), 789-807.

Yang, D. C., Reys, R. E., & Reys, B. J. (2009). Number sense strategies used by preservice teachers in Taiwan. *International Journal of Science and Mathematics Education, 7*(2), 383-403.

Appendix 1. Implementation Examples for the Development of Number Sense

Sample exercises for the "understanding the meanings of numbers" component:

The teacher starts with the following scenario: "Centuries ago, a shepherd encounters a problem in a time when people neither use numbers nor have learned how to count. The shepherd takes his sheep out to his fields to graze and wants to ensure that none are lost while grazing and that they all return to the sheepfold. He can't decide how to do it. He thinks, thinks, and thinks..."

Students read this text as a group and then write suggestion letters to the shepherd. The group spokespersons read the letters which are then discussed. The content of these discussions includes concepts such as how multiplicity can be expressed, the concept of numbers, places where numbers are used, the importance and meaning of numbers, and one-to-one correspondence.

In order to understand the meaning of numbers in the same lesson plan, the teacher hands out paper and writes the following text on each row before asking the students to complete the blanks in the text with the numbers given in the top. In addition, when filling in the blanks, students are told that the numbers they put into the spaces must be meaningful in the given sentence, and they are encouraged to discuss the options with friends.

"When Didem left school, she had to walk .... meters to get home. On the way she passes by .... apartments standing side-by-side. She ponders the height of the 10-story apartment building and estimates it is about .... meters high. Didem believes her bag is very light today as she walks home. She thinks she has a maximum .... kg load in her bag. Didem thinks she may have forgotten her notebooks in school and turns to go back." (Numbers to use: 2, 50, 500, 6).

Here, the students discuss the numerical values of the quantities. They suggest that they can be expressions of magnitude that can be used as symbolic symbols in their daily lives. Students also benefit from their estimation skills in this exercise.

Sample exercises for the "decomposition and recomposition of numbers" component:

The students are presented with a situation in which a 240 x 120 meter garden is divided into a 200 x 120 meter plot and 40 x 120 meters for a second plot. Students are asked to decide whether these two plots are equal to the size of the whole garden.

After conversing in their groups, in order to perform a calculation, such as 23 x 15 = ?, they discuss whether it is possible to use the algorithm 23 = 20 + 3, 23 x 15 = (20 x 15) + (3 x 15). The discussion continues with several different examples and terminates by emphasizing that mental computation becomes very easy by using the decomposition and recomposition numbers strategy.

Sample exercises for the "magnitude of numbers" component:

Students work in groups comprised of three people. First, they discuss the following question:

"Aylin draws a number line in her notebook. Mark the locations of $\frac{2}{5}$ and $\frac{3}{5}$ on the number line. Burak wants Aylin to write another fraction between these two fractions. Aylin says this is not possible because there is no other fraction between the fractions. Do you think Aylin is right? Defend your answer on the number line."

After the groups work among themselves, a class discussion is held where they consider the size of the given fractions. The students question whether they can suggest larger and smaller fractions than the given fractions. Drawings are used in this query. After a summary of the situation is given during the class discussion, the teacher gives each group a piece of rope of the same length (the students ae reminded that one side of the rope represents the number 0 and the other side represents 10). Cards are written with different fractions and natural numbers on them and placed with tack-it fasteners. The teacher wants the students to think of the rope in their hands as a number line and asks them to place the fractions by paying attention to their places. After all groups have completed the placement process, the ropes are fastened to the board and the class discusses how the fractions were ordered on the ropes (i.e., number line).

Again, in an activity to improve *the magnitude of numbers* component, students are divided into four groups. Each group is given a tape measure. The teacher explains the question to be considered. If your height was 10 cm, how many centimeters should your belongings be to fit you properly? The teacher explains that they can use the height of one student and choose objects such as doors, bags, and shoes as objects for comparison. After the measurements and operations are completed, each group discusses the object of their choice, including the real length of the object and the recommended length to be suitable for a 10 cm human. In this way, students are introduced to ratio-proportion concepts while working on number magnitudes.

Sample exercise for the "comparison" component:

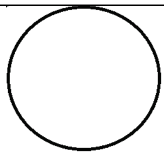
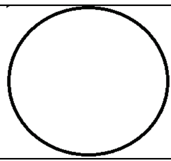
The teacher projects the following operations on the board, with each process visible for only twenty seconds. It is important for students to note whether the results of these fractions are greater than 1 (where 1 is chosen as the reference point here).

$$\frac{1}{8}+\frac{4}{5}, \frac{9}{10}+\frac{7}{8}, \frac{3}{4}-\frac{1}{3}, \frac{11}{12}-\frac{3}{4}, 1\frac{1}{2}-\frac{9}{10}$$

After all the questions have been shown, the teacher returns to the beginning to review each question one by one. Students are asked to explain what and how they think. The strategies used to answer the questions are discussed.

Sample exercises for the "understanding the effects of operations on numbers" component:

The students consider how a different number of divided quantities (fractions) can be combined when adding fractions. This discussion starts from field models regardless of rules or algorithms. The students see how quantities are affected by the process when adding fractions. Similarly, when multiplication is performed in fractions, they see how the whole is affected by the process, as discussed in the following models:

Task	Initial Ouantitiy	Given Fractions of Amount of Initial Quantitiy
Let's find the $\frac{3}{4}$ of $\frac{1}{3}$ of a pizza.		
Let's find the $\frac{9}{10}$ of $\frac{2}{3}$ of a cake.		

Sample exercises for the "flexibility in applying number and operation knowledge to calculation situations" component:

This component includes estimation, mental computation, deciding which calculation tool is most effective and accessible, deciding whether a definite or approximate result will be the appropriate answer to the problem, selecting an appropriate strategy, and testing the significance of the result when solving a problem. These skills are all included in the lesson plans for improving the skills related to this component.

For example, in one exercise, the teacher brings marbles in a jar to the classroom. She explains that the jar cannot be opened; however, she wonders how many balls are in the jar and asks the students to help her. Students use different strategies to estimate the number of marbles in the jar.

In the same lesson, the teacher presents the students with a problem: "How many buses do you need to carry 1,128 students if each school service bus can carry 36 students?" The aim here is to question the students' answer (1,128 ÷ 36 = 31.3) to reveal whether the answer is really a solution to this problem and to think about alternative ways to revise their answers.