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Teaching Probability through Understanding by Design: An Examination on Students' Achievement, Attitude and Views *

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Abstract

In this study, the effects of teaching the 10th grade students the subject of probability through lessons prepared via Understanding by Design (UbD) on their probability success levels and attitudes and also the students' views about this process were examined. The study was designed as an embedded experimental model. The students' attitudes towards probability were measured through the scale developed by Bulut (1994) and their success levels were measured by the achievement test developed within the scope of the study. The students' opinions were collected through individual interviews. Based on the analyses, it can be stated that although the teaching practice increased the experimental group's success level significantly compared to the pretest, it did not make a significant difference in the post-tests compared to the control group; however, it increased the attitude level of the experimental group significantly compared to both the pretest and the control group. Moreover, the students stated positive opinions about the way of teaching the course and its contribution to lessons and also emphasized the importance of probability in decision-making.

Keywords

Understanding by design
Teaching probability
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Mixed methods research

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Introduction

Learning, a process which students are expected to experience at schools, may happen at every moment of life. Resources of learning process, namely individual experiences, social interaction and information access possibilities, increased and varied a lot. In this case, learning processes at school should not only be "desired" and "systematic", but they also need to be of high quality and interesting.

At school, one of the areas in which high quality learning is required to take place is mathematics, a field of occupation which has no exact purpose, but for this very reason, serves many purposes (Nesin, 2003). Although sufficient emphasis is not laid, mathematical concepts have a lot of areas of use in real life. These areas range from the statement of quantities which we use in daily life via numbers to the understanding of economic data and overcoming architectural measurements and calculations at simple and complicated levels. Some mathematical concepts are used for complicated functions in several scientific areas on one hand and have basic but important roles in anytime of the day on the other hand. Probability as one these concepts not only contributes to such fields as management, economics, banking, insurance (Gürbüz, Birgin, & Çatlıoğlu, 2012), genetics, weather

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forecast (Abelson, 1995) and even quantum physics and the chaos theory (Bulut, Ekici, & İşeri, 1999) but also form a basis for reasonable decision making process as one of the most important cognitive activities of real life (Halpern, 2003; Sharma, 2006). Importance of probability is understood better if negative situations which might be caused by the lack of probabilistic thinking are considered. These can be listed as prevention of reasonable thinking, deciding based on preconceptions, the inability to evaluate risks reasonably, being taken with unreasonable fears, having difficulty in making decisions in unclear situations and the inability to interpret statistical data (Nickerson, 2004).

When the benefits of probability in various fields and its effects on the decision-making process which is especially vital for each individual are taken into consideration, it is clear that probability is a subject which individuals should learn and make sense of correctly. However, studies indicate that the teaching of probability is problematic in both our country and other countries (Bulut et al., 1999; Fischbein & Schnarch, 1997; Garfield & Ahlegren, 1988). The mentioned study results revealed that the problems related to the teaching of probability resulted from many different sources. One of the major problems is that students perceive probability as a difficult subject (Green, 1982; Sharma, 2006; Watson & Kelly, 2007). Kutluca and Baki (2009) found that probability was among the difficult 10th grade mathematic course topics according to the opinions of the teachers, students and pre-service teachers. Other sources of the problem can be listed as not using appropriate materials (Gürbüz, 2006), teacher-centered education (Pijls, Dekker, & Van Hout-Wolters, 2007), having problems with probabilistic thinking (Munisamy & Doraisamy, 1998), wrong foreknowledge and beliefs (Sharma, 2006) and misconceptions resulting from incorrect connections established between practical and theoretical knowledge (Gürbüz, 2010).

Most of the studies done on the teaching of probability investigated into misconceptions which students had in the learning process of probability and ways of eliminating these misconceptions (Çelik & Güneş, 2007; Fischbein & Schnarch, 1997; Green, 1982; Gürbüz et al., 2012; Gürbüz & Birgin, 2012; Munisamy & Doraisamy 1998,). In order to overcome the misconceptions related to the subject of probability, suggestions such as using concrete materials in the lesson (Çelik & Güneş, 2007; Gürbüz et al., 2012) and computer-assisted instruction (Gürbüz & Birgin, 2012) can be made.

In the studies done in Turkey on the teaching of probability, in addition to misconceptions, there is also a focus on student achievement. The results revealed that achievement was not at desired level (Bulut & Şahin, 2003; Memnun, 2007). However, finding that the teaching environments based on active learning affected the students' achievement levels with respect to probability in a positive way, for instance, Memnun (2007) suggests that the strategy of learning by discovery and the instruction methods based on play should be used predominantly.

Sharma (2015) states that although there is a rich literature on misconceptions related to probability, students do not show sufficient interest in the development of their probabilistic thinking. In order to provide such an activity and, besides this, create an instruction which will take students' prior knowledge and beliefs into consideration, Sharma proposed a model based on social constructivism. In this model, as teaching concepts and calculations of probability, using probability experiments is suggested and during the process students should make predictions about the experiments and compare the results and their predictions. Moreover, Hay (2014) proposes a model called “dialogue hierarchy” to use in the realization of the process of constructing students' knowledge related to probability with the aim of developing their probabilistic thinking and understanding. In this model, a dialogue starting from the concrete, descriptive characteristics of a problem situation and continuing toward its abstract state is designed and used.

In Vygotsky’s social constructivism, there is social interaction and language, which is the primary means achieving this interaction, underlying learning (1934/1987, as cited in Driscoll, 2005). Learning is realized through the interaction between the learner and another more capable person (teacher, parent, peer, etc.). For, according to Vygotsky, learning occurs in a place between what individuals can do by themselves and what they can do with the help of people who are around them

– which Vygotsky defines as the Zone of Proximal Development (ZPD) (1978, as cited in Schunk, 2011). According to the mentioned opinion, the student should be in interaction with peers as much as the teacher. Then peer interaction should be involved in teaching more effectively. Especially, for students to make meaningful learning, it is thought that using collaborative teaching methods including peer interaction is important.

The present study adopts the idea that the most important aspect of the subject of probability in terms of students is that it contributes to their making reasonable decisions. Based on this thought, making sense of the probability concepts and calculations is important in terms of using these in decision-making situations. Moreover, studies indicate that although students manage to make calculations, they have difficulty in understanding what they mean (Ben-Zvi, 2004; McGatha, Cobb, & McClain, 2002). When the above explanations are considered as a whole, the importance of developing a learning-teaching process based on social constructivism in the teaching of probability, adopting the strategy of learning through discovery and the collaborative learning methods, and focusing on real life in learning and evaluation processes is clear. Moreover, it is considered that an instruction to be done in this way will also contribute to the student's probability achievement in the academic sense.

At the point of providing a learning-teaching process desired to be developed with a certain psychological basis, there is a need for an instructional design to transfer learning and teaching theories or approaches into the teaching process systematically (Driscoll, 2005). In an instructional design, principles put forward by theories are based on and teaching situations which are to guide the teaching process are prepared and put into work (Fer, 2011). Since products, which are practical, directly related to practitioners and even can be developed by them, are introduced within an instructional design and various models guiding the designing process were developed. From among instructional design models, it is very important to choose the most appropriate one for the adopted psychological basis and the near and far objectives of the process.

According to the developers of the "Understanding by Design (UbD)" instructional design model, Wiggins and McTighe (2005), what any design based on understanding should do is not to take phenomena having implicit meanings directly. Instead of this, the design should help students develop awareness about the importance of revealing the implicit meaning and focusing on this. According to theoreticians, this completely corresponds to the constructivism. For it is the same as the constructivist process' claiming that a meaning cannot be taught but can only be created by the learner under the teacher's guidance. It is considered that the UbD, with this aspect, is in harmony with the constructivism, which this study based on as a learning-and-teaching theory, and will be effective in reflecting the theory into learning.

As it is understood from the name which it takes, the UbD is an instructional design model focusing on sense-making and maintaining that making sense of concepts and processes under instruction within the context of real life should be the main goal (Wiggins & McTighe, 2005). In this study, it is projected that designing a probability instruction through a model based on understanding will enable us to observe how making sense of concepts learned and calculations made will take place especially in the real-life context.

The UbD instructional design model is formally a backward design. The model includes the stages of "Identify the desired results", "Determine assessment evidence", "Prepare a learning plan" respectively (Wiggins & McTighe, 2005, p.18). It can be stated that the main reason underlying this form is to determine the place to reach first of all and then make a plan on how to go there. Moreover, the model provides detailed and original instructions and even sub-steps in one sense at every stage. The model's aim of catching the real-life sense-makings of concepts and skills can be observed at every stage. The goal defining process includes a big idea, a real-life acquisition to be reached via academic acquisitions of the instructional process, and a performance, a real-life correspondence of the content, as an evaluation proof; the learning process includes activities which students are likely to encounter in their own lives (Wiggins & McTighe, 2005).

Instruction based on UbD aims to improve students' six facets of understanding, namely explanation, empathy, self-knowledge, perspective, application and interpretation. Along with that all of these skills are important, explanation, perspective and interpretation are directly relevant aims and problem of teaching probability and provide the main motivation for preferring UbD in instruction design of probability. The skill of *explanation* is defined as "To explain a situation, an idea or a case and make meaningful connection, the student can present complicated cause and effect relationships" (Wiggins & McTighe, 2005, p.85) and it is thought that this skill will contribute in coping with issues of inability to make sense probability calculations (Ben-Zvi, 2004; McGatha et al., 2002) and make connections between probability and other concepts (Çakmak & Durmuş, 2015). Furthermore, the two skills *perspective*, defined as "Student can adopt knowledge and theories into contexts and know the problems and questions that could be solved by the knowledge and theories." (Wiggins & McTighe, 2005, p.95) and *interpretation* defined as "Student can interpret parts, data and situations" (Wiggins & McTighe, 2005, p.88) provide insight on usage of probability knowledge in real-life decision making processes. Individuals always make decisions at every passing second of the life and these decisions affect how their life will be. The importance of making these decisions based on particular reasons instead of emotions, intuitions or coincidence is obvious. One of the factors determining the quality of these decisions is knowledge and skill of considering properly the possible outcomes. To sum up, it is thought that UbD is an appropriate model to solve the problem of making sense in teaching probability and realizing the aim of using probability in real life.

Under the light of the above-mentioned explanations, it was decided to use the UbD model in the design for the probability instruction to be implemented within the context of the present study on the grounds that both it is a design model which is in line with the constructivist approach and its main focus is on sense-makings and real-life context. In this direction, the aim of the study is to reveal how a learning-teaching process prepared via the UbD instructional design will affect the 10th grade students' interpretations with respect to the subject of probability, achievement levels and attitudes toward probability. In line with this purpose, the following research questions were determined:

- How does an instruction designed via the UbD affect the students' achievement on the subject of probability?
- How does an instruction designed via the UbD change the students' making sense of the subject of probability?
- How does an instruction designed via the UbD affect the students' attitudes toward the subject of probability?
- What are the students' opinions about an instruction designed via the UbD?

Method

The Research Design

In the study, the mixed method in which qualitative and quantitative data were used together was adopted. The designs employing the mixed method are classified differently according to different authors. In this study, it is aimed to use both quantitative data including achievement, attitude scores and qualitative data in which views are stated and sense makings show up in order to answer research questions. Also, to understand to what extend the instruction designed is effective and to observe students' development, it needs to set up an experiment for the former and to collect data during the process for the latter. The embedded experimental model, one of the research methods employing the mixed method according to the classification made by Creswell and Clark (2006), is taken as the most appropriate model for the aims and requirements of the study. However, because in public schools it is not possible for researchers to assign students into classes randomly, experiment and control groups are randomly assigned to existing classes. Therefore, the design of the present study was determined as embedded semi-experimental design. This model can be schematized as follows.

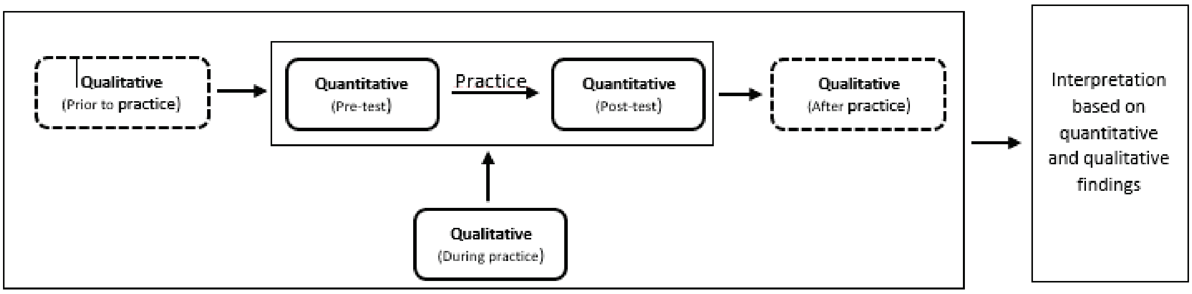


Figure 1. The Embedded Experimental Model Used in the Study

The following steps were followed during the study:

- Firstly, in order to determine the instructional need, a needs analysis was made by taking teachers' and students' opinions and reviewing the related literature. As a result of this, it was decided to teach the subject of probability through the adoption of the constructivist approach and the strategies of learning by discovery.
- In the design of the instruction, the UbD instructional design was used. By following the steps of the model, the desired results, the evaluation proofs and the teaching plan were constructed by the two researchers, one of whom was the teacher of the practice.
- An achievement test was developed and applied to the students attending 10th grade mathematics course at the same school before.
- The students to participate in the instructional practice were informed about the aims of the study and the achievement test and the attitude scale were administered as pretest. The students' learning styles were determined through the related scale.
- The learning plan was administered by the mathematics teacher, one of the researchers. The practice lasted 14 hours except for the performance task and the administrations of the tests. During the practice, the qualitative data was collected through open-ended questions and performance task from the students. With the aim of evaluating the practice, tape recording was made during the practice and 2 hours of observation was made by the other researcher.
- The achievement test and the attitude scale were administered as post-test. The students were asked for their opinions through individual interviews.

The Study Group

The study group was composed of the 10th grade students of an Anatolia High School in Istanbul. The school, where one of the researchers worked as a mathematics teacher, was selected through the appropriate sampling method. One of the three classes of the school was determined randomly as the experimental group to which the instruction would be given via the design and another class was selected as the control group. The math lessons of both classes were taught by the same teacher. Summary information about the study group is presented in table 1.

Table 1. Descriptive Data about the Study Group

Group	Grade Level	Class Size	Gender Distribution	Absenteeism (Mean)	Previous Year Achievement Score (Mean)
Experimental	10^{th} (15-17 years)	27	14 female, 13 male	6	56.1 out of 100
Control	10^{th} (15-17 years)	25	11 female, 14 male	5	56.8 out of 100

As it can be seen in table 1 both the class size and gender distributions of the experimental and control groups are similar. Previous year (9^{th} grade) mathematics achievement levels of students from each group are also similar. These are considered important in terms of group homogeneity.

Design Process

Desired Results: The study began with a needs analysis. The literature was reviewed in relation to the subjects which students had difficulty in understanding in math education. Moreover, two math teachers were asked for their opinions. From these resources, it was concluded that probability was one of the difficult subjects in the secondary math education. With the aim of including the learner analysis in the needs analysis, interviews were held with four 11th grade students, two with low success level and two with high success level. The students stated having difficulty in learning the subject of probability. Moreover, they answered the questions related to how the subject of probability should be taught. To form experimental and control groups of the study the 10th grade students were administered an exam composed of open-ended questions and measuring the prior knowledge for probability. It was determined that the students were not sufficient at the 9th grade probability topics which constitute the basis for the 10th grade probability topics and the situation was the same at the counting topics which are necessary as prior knowledge. Here, it is thought that 10th grade counting topic should also be included in the instruction based on UbD because it contains basic pre-requisite learnings for 10th grade probability topic. As a result, it was decided to make a design covering 9th grade probability, 10th grade counting and 10th grade probability topics.

The instructional design was started by taking the data obtained from the needs analysis and the learning objectives of the related topics were prepared in line with MNE. Firstly, in the scope of the desired results "general aims, big idea, essential questions, knowledge statements and skill statements" which are pre-constructed by Wiggins and McTighe (2005, s.22) were determined. These are presented in table 2.

Table 2. The Desired Results

General Aims: I is aimed to Explain basic concepts about probability, Use probabilistic calculation methods and Calculate probability of simple, independent and combined events	
Big Idea: Importance of using probability knowledge in correct decision making	**Essential Questions:** Why do we learn probability? What does that probability of an event is high/low mean? How events' affecting from each other reflects on their probabilities? Is it possible to calculate probabilities of events' that will take place respectively and one will affect the other?
Knowledge Statements: The students will be able to know *Terms of* experiment, event, universal set *Concepts of* complement of an event, mutually exclusive and non-mutually exclusive events, certain event, impossible event, independent event, combined event *Operations of* factorial, permutation, combination Judgment of probability can take values between 0 and 1.	**Skill Statements:** The students will be able to Write universal set of an event, Calculate probability of a simple event, Calculate probability of the complement, Compare probabilities of mutually exclusive events, Calculate the number of events' taking place, using permutation and combination Calculate probability of an event with a given condition Determine whether the events are dependent or independent Calculate probability of independent events Calculate probability of dependent events.

Within the scope of the designed instruction, with the knowledge and skills they have acquired, the main point the students were expected to reach is to comprehend the importance of probability knowledge and use it in decision making correctly.

Evidence of Evaluation: It was decided to obtain pieces of evidence, which were determined to evaluate the level of reaching desired results, through three open-ended exams and a performance task. Of these, the open-ended exams were the individual work products and the performance task was the group work product. More detailed information is presented in the data collection tools part.

Learning Plan: Finally, eight two-hour learning plans (total of 16-hour) were prepared as detailed lesson plans. One example of the lesson plans is presented in table 3.

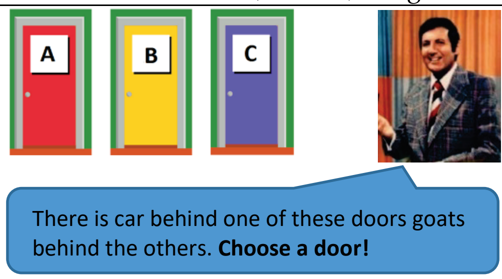
Table 3. Summary Information about Lesson Plan – 2

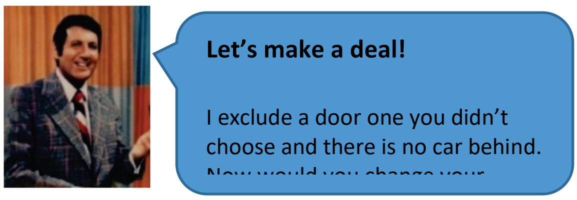
Unit/Topic: Data, Counting and Probability / Simple Probability. **Grade level:** 10. **Duration:** 2 hours

Objectives: - Calculate probability of a simple event. - Calculate probability of the complement. - Calculate probability of mutually exclusive and non-mutually exclusive events.

Method-Technique: Discovery – Question & answer **Material:** Work sheet, videos, images.

Intro. – Getting attention: Monty Hall problem is presented. (On the right)
Process: Students are asked what they would have done. (the lesson goes on following work sheet.
-The probability at first case is calculated.
-Second image is presented (below)





-Students are asked if they change their previous choice and this issue is discussed.
- They are expected and motivated (but not said directly) to think how the probability is change when they change the door.

- Individual decisions about changing or not are noted. (Either on worksheet or on the board)
- The door, the car is behind, is made public.
- Number of the winners from decision changers and not changers are examined and discussed.
- A video is watched. (From a movie a scene including Monty hall problem)
- Students are asked to write what they reach in this activity, on work sheet.

Students are asked to solve the other examples presented on the worksheet.

The second activity is implemented. For details of the second activity the work sheet presented in the appendix can be seen.

Evaluation: An open ended exam will be implemented for basic concepts and simple probability.

Two-hour lesson plan presented in table 3 is on simple probability. Objectives of the lesson are calculating probability of a simple event, the complement, mutually exclusive and non-mutually exclusive events and evaluation of them is based on an open ended exam. Learning process includes presenting Monty Hall problem as a game, working on the problem with the help of teacher explanations and additional materials (video) and using simple probability and complement in this case.

Implementation Process

The implementation process of the design can be summarized as follows.

- The 16-hour design was planned and administered in seven 2-hour lessons and 2-hour performance in the end.

- As it can be seen above, each lesson was started with activities such as discussion, watching and commenting on related videos and carrying out different probability experiments with the help of concrete materials.
- The lessons continued with activities including real-life situations, probability experiments, questions related to these and asking the students to discover the points on which the lessons focused. After that, some examples were examined in relation to the knowledge and skills which the students were required to reach.
- In these essential activities, the students used the worksheets prepared separately for each lesson. In the lessons, all the works except for the evaluations were performed as pair work. Two students working on the same task were asked to use different colored pencils so as to see if both students were active in the tasks. The samples of the worksheets completed by the students and a working scene is presented below in figure 2.

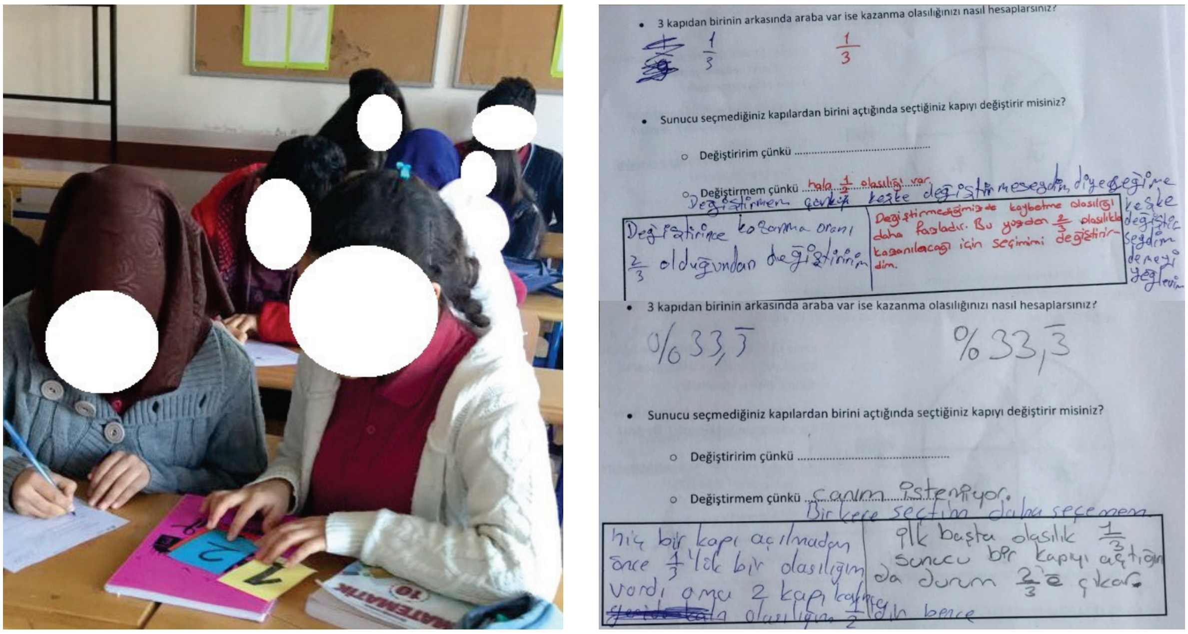


Figure 2. Displays of a Pair Work Scene and Works sheets.

- While the above design was implemented in experimental group, in the control group direct instruction, question and answer methods and problem-solving techniques were used. However, in this group real-life situations, probability experiments with concrete materials and worksheets used in experimental group were not used. Problem solving and examples were based on written materials. The examples solved in experimental group and additional examples were solved in control group. Because open-ended exams are part of the design they were not used in the control group, only achievement test and attitude scale were administered.

Data Collection Tools.

The research data collection process is summarized in table 4:

Table 4. Data Collection Process

Variable	**Data Collection**				**Analysis**
	Data	**Tool**	**Group**	**Design**	
Achievement: Learning concepts and making calculations in probability topics	Achievement score	Achiev. Test (Developed)	Experimtl. & Control	Pretest & Posttest	Covariance Analysis
	Opinions	Individual interview	Exp.	Post T. 10 prs.	Descriptive Analysis
	Concepts, calculations	Open-Ended Ex.	Exp.	Process-indv.	
		Performance Task	Exp.	Post T-group	
Sense-making: On decision making process and calculations	Opinions	Individual interview	Exp.	Post T. 10 prs.	Descriptive Analysis
	Explanations	Open-Ended Ex.	Exp.	Process-indv.	
		Performance Task	Exp.	Post T-group	
Attitude: Feelings and behaviors related to probability topic.	Attitude score	Probability Attit. Sca. (Bulut, 1994)	Experimtl. & Control	Pretest & Posttest	Covariance Analysis
	Opinions	Individual interview	Exp.	Post T. 10 prs.	Descrip. A.
Opinions: Related to the instruction based on UbD	Opinions	Individual interview	Exp.	Post T. 10 prs.	Descrip. A.

Achievement Test: In the present study to determine students' achievement levels on probability, a 46 - item - test based on objectives of 9th and 10th grade probability topics and also the 10th grade counting topics as prior knowledge has been prepared.

Three experts from the fields of mathematics teaching, measurement and evaluation and instructional design respectively were asked for their opinions and, based on these opinions some items were reviewed and the number of items was decreased to 40. The 40-item test was administered to the upper class students having previously studied the relevant subjects at the same school and the administration results were analyzed through the help of the Classical Item and Test Analysis Spreadsheets (CITAS) software (CITAS, 2015). The KR-20 reliability of the test was determined as 0.90 and the average item difficulty was calculated as 0.50. According to the item analysis, the questions with an item discrimination of below 0.20 were omitted; some of the questions with an item discrimination of 0.20-0.30 were omitted; small corrections were made in the sentence structures and some modifications were made on the distracters of the items which had to remain; as a result, an achievement test of 30 items was obtained. Each of first four items based on concept recognition scored 2 points and each of other items scored 3,5 points. The test's KR-20 reliability coefficient obtained from the scores of the test applied as the post-test in the study was 0.88. Test's item distribution over the units and item examples are presented in table 5.

Table 5. Achievement Test's Item Distribution and Item Examples

Unit / Topic	Objective	Number of items	Item examples
Probability / Simple Probability	Explain basic concepts, calculate probability of simple events.	10	What is the probability of drawing the ball that is not red from the bag in which there are 3 red, 3 blue, 5 black balls?
			Which one of the below cannot be probability value of an event? A)0 B)1/2 C)3/4 D) 1 E) 4/3
Counting: Sequence and Selection	Calculate number of different cases (different sequences and selections) in an event.	10	A three person commission will be formed among 4 mathematics and 3 chemistry teachers. What many different commissions can be formed?
Probability: Conditional Probability	Calculate probability of an event with a given condition. Determine whether the events are dependent or independent. Calculate probability of independent events and combined events.	10	A man has five different colored trousers including a yellow one and four different colored shirts including a navy-blue one. What is the probability of his wearing the yellow trousers and navy-blue shirt?
			In a class with 40 students, 8 of 16 girls, 6 of 24 boys are wearing glasses. If a student selected from this class is a boy, what is the probability of his wearing glasses?
			There are 3 yellow 5 purple in bag A and 2 yellow 3 purple in bag B. A dice is rolled. If it comes up 3 or smaller, a ball from bag A and if not a ball from bag B is drawn. What is the probability of the ball drawn is yellow?

As it can be seen above, the achievement test was to measure level of reaching to objectives presented in the national curriculum and type of questions in the test were those of the official textbooks and high-stake tests. Here the aim was to measure academic achievement with the traditional meaning and to examine effect of the instruction on this type of achievement.

Probability Attitude Scale: The students' attitudes toward the subject of probability were determined with the help of the Probability Attitude Scale developed by Bulut (1994). The scale was composed of 28 items and the total score ranged between 28 and 168. In the analyses made in the development process of the scale, it was observed that the scale had one dimension and its Cronbach's alpha reliability coefficient was 0.95. The original scale was developed using 8th graders data. That is why, a confirmation from the developer of the original scale was asked and received to use it for 10th graders. In this study, the Probability Attitude Scale was administered to the experimental and control groups as the pretest and post-test and the reliability coefficients were calculated as 0.91 and 0.95, respectively.

Learning Styles Inventory: In the study, with the "Learning Styles Inventory" developed by Grasha (1982) and adapted into Turkish by Zereyak (2006) (Cronbach's alpha: 0.83), the students' learning styles were determined. The data related to the students' learning styles was used in the formation of groups for performance task. It was not included in the analysis.

Open-Ended Exams: In this study, after each of simple probability, counting and conditional probability topics, an open-ended exam about the topic was administered. Aim of these exams was to reveal students' sense-making on concepts and calculations of probability they used. Therefore, rather than a quantitative scoring, analyzing and observing their sense-makings was at the forefront.

The exams, in which students performed individually, consisted of questions situated in a particular context. A review of the open-ended exam administered after simple probability topic is presented in table 6.

Table 6. Reviewing the Open Ended Exam for Simple Probability Topic

The Context	Questions	The Goal
Students were given a coffee menu in a language they cannot speak and they are asked to order a coffee.	"Write a set consists of the all possible orders" "What is the name of this set?"	Explaining and using the basic concepts: Universal set.
	"Assume that there are 5 coffee you would love, what is the probability of you love the coffee you ordered randomly? What is the probability of you do not love the coffee you ordered?	Using the basic calculations of probability: -Probability of a simple event -Probability of a complementary event
	Is it reasonable to order a coffee? Why?	Using probability in correct decision, justifying the decision by probability knowledge.

As it can be seen in table 6, students are placed in a problematic situation in a real-life context (coffee order from a menu in a foreign language they do not understand). In this situation they are expected to integrate their probability related knowledge and skills into the decision making process which is an important real-life process and to make sense of this process. As preparing these exams, researchers received expert opinion from teaching mathematics and instructional design experts respectively. Full texts of the exams are presented in Appendix 2.

Performance Task: An evaluation in which students, in a given context, pretend some role, produce something in some standards for a goal and present and defend this production, is named as performance task (Wiggins & McTighe, 2005). In the present study, performance task required students to use probability knowledge and skills in a real-life situation. The students were asked to tell the customers in a real estate agency that rank is not important in an estate drawing by using probability calculations. For this explanation, they were supposed to prepare a poster. The task required guiding a decision-making process to be realized by using probability knowledge. The standards of the performance task were determined as the students' being able to use probability calculation knowledge and skills effectively in the decision-making process and concretize the meaning and interpretation of calculations through visualization for other people. It is considered that the task will make a contribution at the point of both making sense of what is learned and developing social skills. The performance task was administered in a group work. The groups were formed in a way to assign the students having the same learning style to the same group at the rate of variety in a classroom. The administration form of the performance task is given in the Appendix 3.

Individual Interview: As an effective method to reveal individuals' views, experiences and feelings (Yıldırım and Şimşek, 2011), individual interview, in this study, was used to determine students' views and feelings about designed instruction. Six questions were prepared and three experts specialized in their fields were asked for their opinions about the questions. "What do you think about the probability lessons we made?" "How did you feel in the lessons?" "How do you compare your participation with our mathematics lessons?" "Are the topics covered in the lessons are useful in real life? How?" "What do you think about importance of probability in decision making?" "What do you think about the effect of this type of teaching on your learning probability?" The first three of the questions focused on the students' feelings and thoughts about the teaching methods and the last three of them focused on if the students reached the goals included within the scope of the design. According to the course of interviews, the questions were asked separately or one within another and, when necessary, probe questions were used. With the aim of increasing the diversity of the data, students to be interviewed were selected according to the results of achievement test, attitude scale and open-ended exams. In the end 25 – 30 minutes interviews were made with 10 students.

Data Analysis

In the quantitative data analysis, to understand how teaching with UbD affected the attitude and achievement levels, the post-test scores belonging to the experimental and control groups' attitude and achievement variables were compared by using ANCOVA. In this analysis, the pretest scores belonging to the same variables were taken under control as covariate. Hence, it was aimed to eliminate the effect of the pretest scores of the groups, that is to say, their status prior to the practice, on the post-test scores (Büyüköztürk, 2011). For both attitude and achievement, the groups' initial levels were controlled, it was possible to compare their final levels more correctly. Thus, more accurate results were reached in relation to the effect of the applied procedure which is the designed instruction on achievement and attitude.

The preconditions for the application of ANCOVA with which the experimental and the control groups' pretest scores can be taken under control and their post-test scores can be compared, are listed as the groups' being independent from one another, the groups' independent variable, that is to say, post-test, scores' showing normal distribution, their variances' being equal and lastly the groups' regression coefficients (slopes of the regression lines) being equal (Can, 2013).

Since the experimental and the control groups were different classes, it can be directly stated that the groups were independent. For the normal distribution of the pretest and post-test scores, the Shapiro-Wilk Normality Test was applied; for the equality of the groups' variances in the post-tests, the Levene's Test was applied; for the equality of the slopes of the regression lines in the groups, the customized model was used. Results from these tests are presented in table 7.

Table 7. Results Related to the Preconditions for ANCOVA for the Achievement and Attitude Variables

Test	Group	p-value			
		Achievement		Attitude	
		Pre-test	Post-Test	Pre-test	Post-test
Shapiro-Wilk Normality Test	Experimental Group	.327	.077	.928	.270
	Control Group	.569	.853	.112	.728
Levene's Test	Experimental - Control	.246		.086	
Difference in Regression Line Slopes	Group*Achievement Pretest	.276		.306	

When table 7 is examined, it can be stated that, according to the Shapiro Wilk Test results, the groups' pretest and post-test achievement scores and attitude scores show a normal distribution at the significance level of 0.05. Moreover, for the both of achievement and attitude variables, the Levene's Test indicates that there is not a significant difference between the experimental and the control groups' variances, in other words, they are equal. Finally, there is not a significant difference between the slopes of the regression line for the experimental and the control groups' achievement scores and attitude scores. Thus, it can be said that both of the variables fulfilled all the pre-requirements for ANCOVA.

In the qualitative data analysis, on the other hand, descriptive analysis was employed to the data collected through the interviews. The exemplary table that includes explanations about qualitative analysis and presentation of qualitative findings is presented in figure 3.

Exemplary Table

Theme	Explanation	Sub-Theme	Code	Data Resource	Frequency
Themes are formed by collecting subthemes under the research questions	Relations between themes and subthemes or data are explained.	Sub-themes are formed by collecting similar codes together.	Codes are main findings obtained from interviews, open ended exams (OEE) and performance task via descriptive analysis.	I: Data from the individual interviews. m1: 1st male student. f1: 1st female student.	Number of students are used as data source in the individual interviews
			In the analysis of OEE and performance task, to what extend the calculations are true and what kind of sense-making the explanations include are determined.	OEE 1 and 3: Data from 1st and 3rd open ended exams	
				Data from **performance task.**	

Figure 3. The Exemplary Table on Qualitative Data Analysis and Presentation of Findings

The students' opinions were gathered under themes and sub-themes and the opinions were stated in tables with their frequencies. The tables were prepared by fundamentally focusing on individual interviews. However, the themes and codes were generated completely from the participants' statements in the interviews. From this respect, it can be stated that the analysis made is in the form of "coding made in accordance with concepts derived from data" stated by Yıldırım and Şimşek (2011). Coding of the interview data was made by two researchers separately and emerging themes and codes compared and determined differences were accommodated. Finally, about consistency among data, code and theme, opinion received from an expert out of this research. Two of the open-ended questions – the first and the last exam – were firstly evaluated academically, more specifically, if the students followed the correct ways and reached the correct results and interpretations according to the type of the question were checked. After that, the general status in the class exams, the students' interpretations and how these interpretations change were interpreted with the help of the students' works. Similarly, the poster obtained as a result of the performance task performed in groups was interpreted in terms of the students' probability skills and perceptions.

For the purpose of developing the validity and reliability characteristics of the study, various measures were taken. Firstly, since the study was built according to the mixed design, it has a methodological triangulation contributing to validity (Cohen, Manion, & Morrison, 2007). This allows for the interpretation of qualitative and quantitative data together and their correcting each other. In the quantitative dimension of the study, the data collection tools, whose reliability and validity had been achieved, were used and the measures related to the reliability and validity of the achievement test developed within the scope of the study were mentioned above. Moreover, in the qualitative dimension, since different data resources such as open-ended exam, individual interview and performance task were used, the data diversity (Patton, 2002) contributed to the validity of the qualitative dimension of the study. Furthermore, in the analysis of the qualitative data, including detailed descriptions such as quotations from the students' expressions in the interviews, images from the open-ended exam sheets and the poster contributed to the external reliability and validity of the study (Yıldırım & Şimşek, 2011).

Results

The findings of the study are presented under the research questions.

1. Findings related to the Students' Achievement on the Subject of Probability

In the beginning, to show how the instruction designed via UbD affects students' achievements on probability subject, findings from achievement test, individual interviews, open ended exams and performance test will be addressed. Firstly, the group means related to the achievement test scores obtained by the experimental and the control groups in the pretests and the post-tests are presented in Table 8.

Table 8. The Experimental and the Control Groups' Achievement Score Means

	Pretest			Post-test		
	Number of Stu.	**Mean**	**Standard Deviation**	**Number of Stu.**	**Mean**	**Standard Deviation**
Experimental	27	33.4	16.8	27	55.2	21.3
Control Group	25	23.2	12.3	22	43.2	20.4

When table 8 is examined, it is observed that the experimental and the control groups' achievement test score means increased in the post-test compared to the pretest. The groups' pretest score means are different, too. The difference between the post-test score means is higher than the one which is between the pretests. To determine if the difference between post-test achievement score means is significant, the results of covariance analysis (ANCOVA), in which pre-test achievement scores are controlled, are presented in table 9.

Table 9. ANCOVA Results of the Experimental and the Control Groups' Post-Test Achievement Score Means

Group	Test	Corrected Mean	Difference of Means	Standard Deviation	p-value
Experimental	Post-test	47.6	-4.0	5.3	.450
Control	Post- test	51.6			

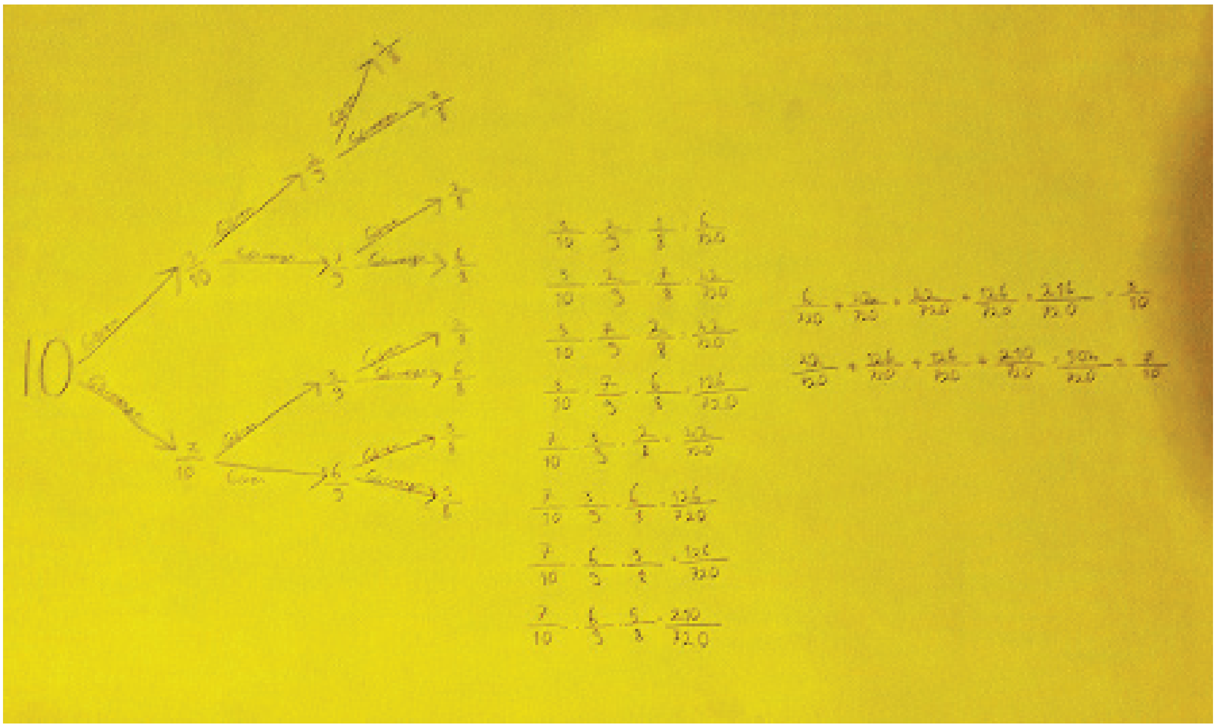
When table 9 is examined, it is observed that there is not a significant difference between the experimental and the control groups' post-test achievement score means. From this point on, it can be stated that the procedure applied to the experimental group did not change their achievement scores significantly.

To get more information about students' achievement on probability subject, data from student views about teaching-learning process, open-ended exams (OEE) and performance task can be examined.

Table 10. Views about Teaching-learning Process, Individual

Theme	Explanation	Sub-Theme	Code		Data Resource	Frequ.
Academic learning	*If students have learned the concepts and calculations*	Understanding the topic	Better understanding of the subject of probability		I: m1,2,3,5,6, f1,2,3,4	9
			Those which help better understanding	I: m2,3,f2	G: e2,e3,k2	3
				I: m5,6	G: e5,e6	2
				I: m5	G: e5	1
		Indv. exam performance	More learning about the concepts Making complete calculations less frequently		OEE. 1 and 3	
		Group performance	Making academic part completely or great completely to a great extent		Performance task	

In OEE-1, students were to do a probability experiment by ordering a coffee from a menu they cannot understand. First, they would have written the universal set of the experiment and calculate the probability of ordering a coffee they like. Most of the students correctly wrote the universal set but less made the calculations. OEE-3 needed to solve a probability problem given in a game of chance context and make a judgment. In this exam, in terms of making calculations student were separated into three groups: ones made the calculation, partially made the calculation and could not make the calculation. When two exams were compared, it is understood that students were better on learning the concepts but showed lower performance on making more complex calculations. Students used difficult probability calculations in order to explain that order is not important in the estate drawing. Examples from the calculations in the posters which are product of group work.



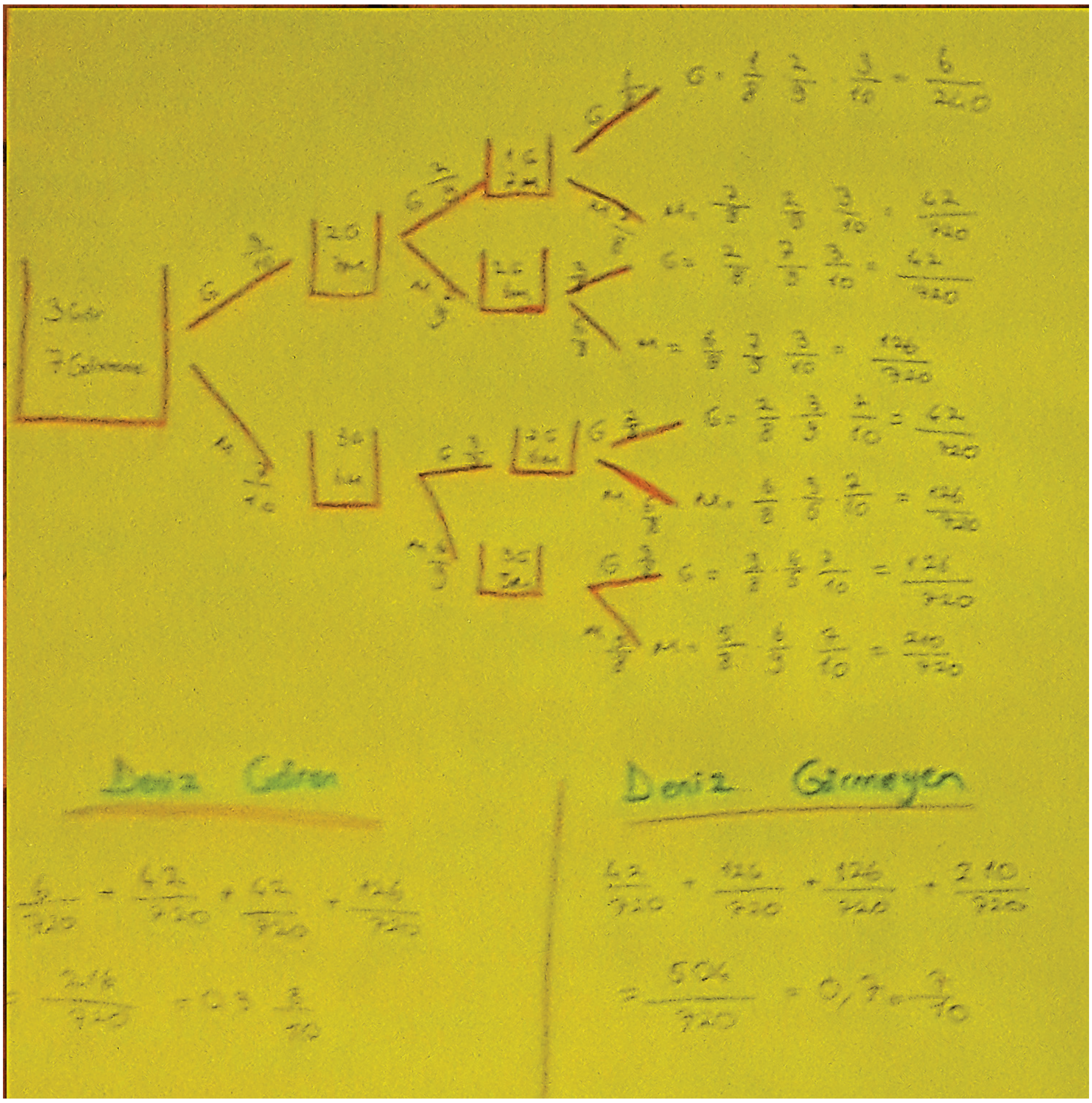


Figure 4. Correct and Incorrect Calculations in Performance Task

When the calculations in figure 4 are examined, it is seen that while some groups made the calculations correctly, others did not. It is observed that students were having trouble on complicated calculations. Expertise on this kind of calculations seems possible via learning question types, formulations and solving number of examples which are common techniques used in traditional teaching of probability as so control group of this study. However, value or return of this expertise in real life is arguable. Moreover, nearly all of the students stated having understood and learned the subject well. When the students are asked about the reason, three of them stated that performing activities helped; two of them stated that the lessons were simple and understandable; one of them stated that since they were given sufficient time in the lessons, they could work sufficiently.

> M3: "We did so many things, but we could learn more than normal. It was better."

> F1: "I normally could not do probability. I was always confused. I could not do it. But since we did so, I could understand probability better. Before the instruction, I could not understand probability at all, but I could understand others. I mean I learned this and I understood these, too."

> M5: "For we worked on it. We worked for a long time. Since we solved it, it was in front of us and you gave us sufficient time, we did it better."

That a great majority (90%) of the students stated having understood the lesson better is very important in terms of the effectiveness of the design. However, not every student could state the reason for their better understanding in the same clearness. Even so, that students think that they understood the topic is seen important in terms of rising their self-efficacy levels.

2. Findings related to the Students' Making Sense of the Subject of Probability

Secondly, to reveal how the instruction via design affects students' sense making, data from individual interviews, open-ended exams and performance test will be discussed. Findings of the individual interviews are presented in table 11.

Table 11. Findings related to Students Sense Makings through subject of Probability

Theme	**Explan.**	**Sub-Theme**	**Code**		**Data Resource**	**Freque.**
Sense-makings	*Students' sense-makings related to decision-making making sense of the calculations which they make*	Probability is important in decision-making			Everyone	10
		Explaining the importance of probability in the decision-making process	Reaching a correct result through probability		I: m3	1
			Thinking on alternatives		I: f1	1
			Thinking on results		I: m4	1
			Multiple thinking instead of single thinking		I: f3	1
			Needing some information when thinking		I: m2	1
			Decision based on feelings and psychology when probability is not known		I: m2,f1,3	3
		Basing their decisions on probability more frequently			OEE 1-3 Perf. Task	
		In real life	Probability is used		I:m2,4,5,6; f1,3,4	7
			Exemplification	Lesson-based	I: m2,4,f1,4	4
				Non-lesson-based	I: m5,6	2
		Making sense of calculations	Gradually more sense-making		OEE 1-3	
			Every student is not at the same level			
			Not directly related to making a calculation correctly		OEE – PT	

As it is seen, all of the students stated positive opinions about the importance of probability in decision-making which is the big idea of the design. However, it is not possible to state that the students put forward quality opinions when explaining their positive opinions. Despite this, five students stated their own opinions about the importance of probability in decision-making. One of these was that a correct result is obtained through probability and this result should be used in decision-making.

> M3: "Probability is important in making reasonable decisions. When we apply probability, we get a result; all of them are equal, but that's all. It is a chance, but we get a reasonable thing with probability. This is normally very non-sense. I mean, if we do not consider probability, it is not reasonable."

Here, it can be stated that student associated importance of probability in decision making with probability's providing a way for most situations generally explained by chance and also explaining the chance to some extent. However, in the other two opinions, the importance of probability in decision-making is explained through its alternatives and results. These opinions were stated by the students in this way:

> F1: "Now, in the journal, to tell the truth, there are many things on which we decided. To tell the truth, there are many things which we did and we did not consider whether to do these or not, we did that way or another, it is probability, there are four alternatives in front of us, for example, this is a probability. Choosing … For example, if I chose this, that is one possibility."

> M4: "For if we are to decide on something, we should learn its results. For this reason, they are its probability. That's why, we make a reasonable decision, for example."

Finally, the students stated that they did not know the probability or when they do not take probability into consideration, they tend to make the decision which they think or feel right. However, this decision may not always bring correct results.

> M2: "I do not know, for example, in that door case, I do not know, I never thought, for example, in that 33% case, in that case, I never thought. I thought it would try to deceive me; it is conflicting with my reasoning. Anyway, there I said I would not change the door. The idea that I will change it after I have learned is conflicting with my reasoning."

> F1: "For example, well, there was something, a quiz. There my thought was the opposite. To tell the truth, for example, if I joined a quiz show, I would increase my chance of winning through probability."

Again, a great majority of the students' statements points out that they think probability is used in real life. However, when they were asked to give examples, few of them managed to put forward examples not given in the lesson.

The sense-makings which the students made in relation to the probability concepts or the subject of probability in general indicate that they inclined toward the big idea of the design and some were ahead of others in reaching this idea. On the other hand, it is considered that it is important to try to have more students make right sense-makings.

In OEE-1, students' tendency of base their decisions about coffee order on probability was not directly related with students' correctly making calculations and small number of students based their decisions on probability.

In OEE-3 students' needed to make a judgement about if the chance game is fair or not and justify this decision, at the end students showed different performances in terms of making sense of calculations. The images in figure 5 include the explanations about the students' judgements.

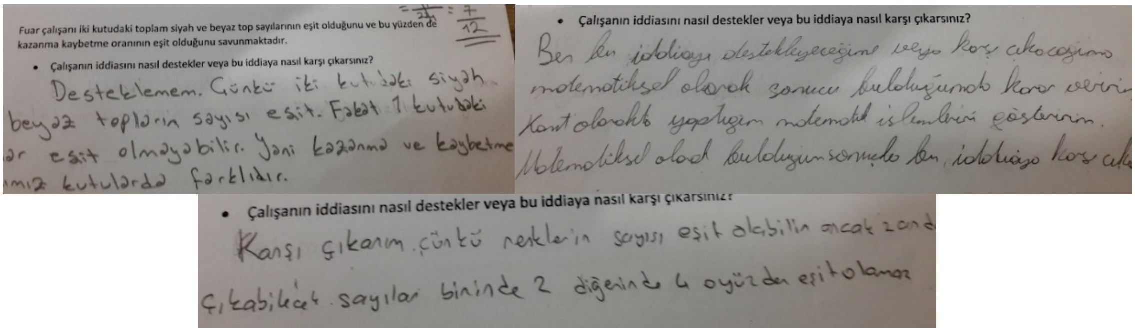


Figure 5. Displays of Open Ended Exam Papers (Students answering correctly)[4]

As it can be seen, in figure 5 students rejecting the claim that game is fair, presenting justifications based on probability. It observed that students, make the calculation correctly by following correct steps, make true judgments about the situation and explain the steps making sense of calculations. The examples from their judgements and explanations are shown in figure 6.

[4] Writings in the figure 4: "I don't support. Because, number of black and white balls are equal in two boxes. However, balls in one box may not be equal. I mean the chance of winning or losing in each boxes are different."" I reject because although number of colors are equal, the numbers will come up in dice rolling is 2 in one and 4 in the other so cannot be equal. " "I decide to support or reject this claim when I find the mathematical result. I show the mathematical process as evidence. Considering the mathematical result I found, I reject this claim."

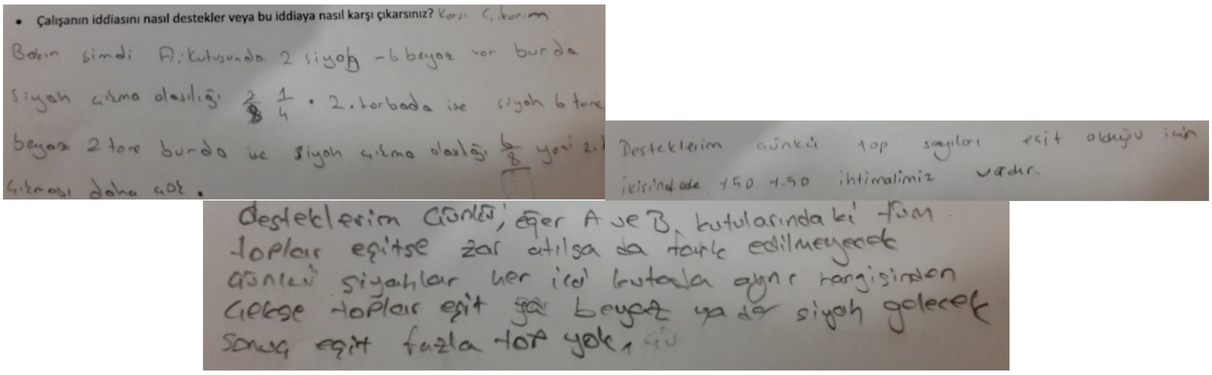


Figure 6. Displays of Open Ended Exam Papers (Students answering incorrectly)[5]

The reason why these students made a wrong judgement was that they could not combine the events in the problem, but they thought they were separate simple events. Mistakes in calculations caused incorrect judgments. Still, it can be stated that students have a correct understanding in terms of making judgments based on probability.

In conclusion, even in the combined probability, which is the top level subject for the high school level of probability, when the students are asked correct questions, it is possible that they make sense of calculations which they make, make judgements by using these and support these judgements. On the other hand, it was observed that the students who could not reach all the academic goals, that is to say, make all the calculations correctly, reached sense-makings over the academic goal which they could attain or had problems in making sense, but the most importantly, generally made wrong judgements.

The most important difference between the two exams was that more students based their decisions and judgements on probability in the second exam. Moreover, again, in the second exam, those who made the probability calculation correctly based their judgements on probability more frequently. Here the striking point is that students' tendency of including probability into decision making processes was improved during the instruction.

In the performance task students are to make poster presentation to the customers of an estate agency. Aim of this presentation is to help customers to understand that order of drawing does not change the probability of getting a good flat. Each group need to provide a justification based on probability calculations and present this through a poster. The exemplary explanations from posters are shown in figure 7.

[5] Writings in the figure 5: "I reject. Now, in box A, there are 2 black and 6 white. Here probability of drawing black is ¼. In second box, on the other hand, 6 black and 2 white. Here probability of drawing a black is 6/8. So drawing from the second more possible." "I support because the number of balls are equal in both so we have 50% 50%possibility." "I support because if the all balls in box A and box B, rolling a dice will make no difference because blacks are the same in both boxes. Drawing from either the boxes, because the balls are equal either white or black will come. The results are the same no extra balls."

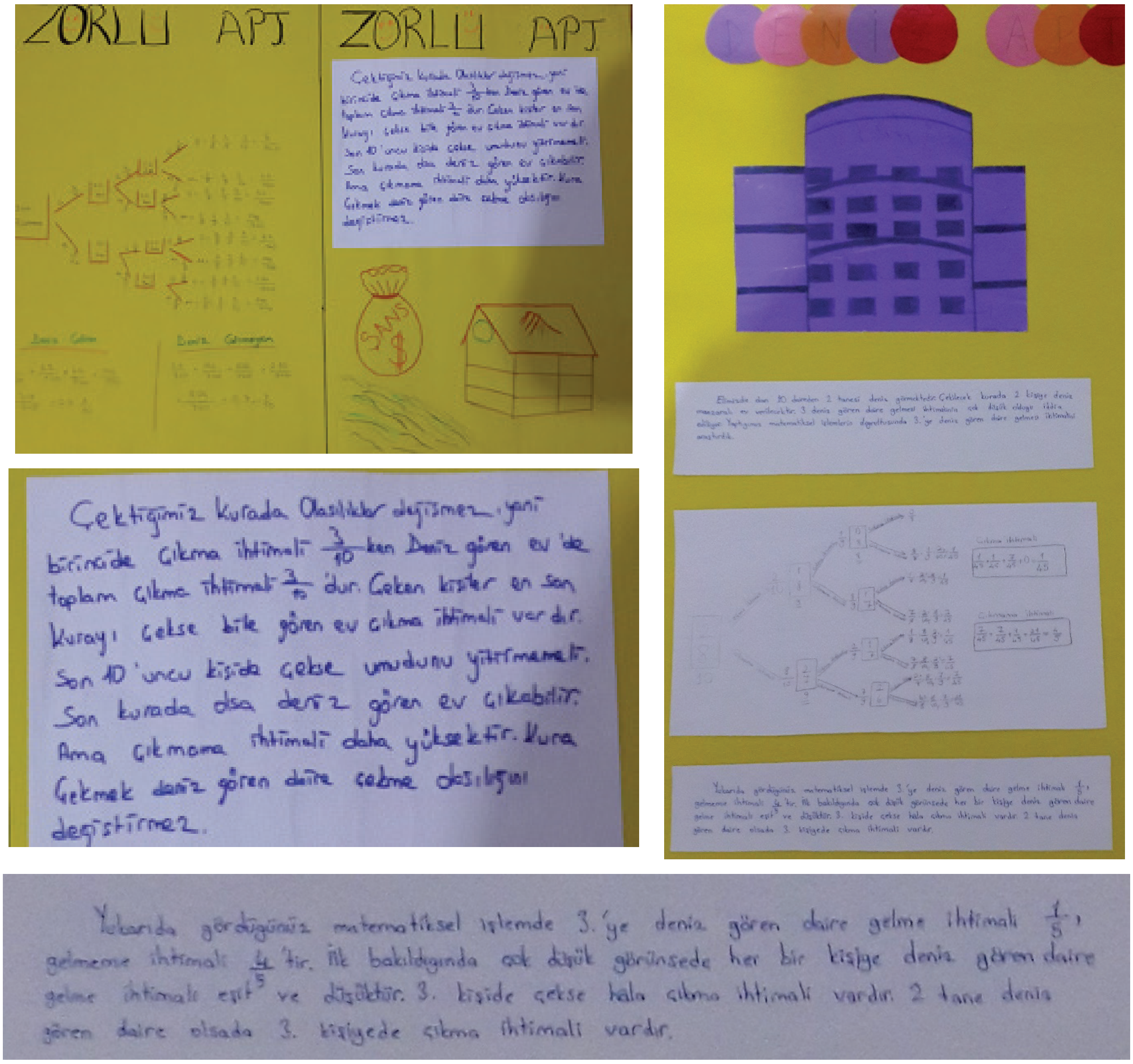


Figure 7. Displays of Posters Produced in the Performance Task [6]

In figure 7, two different posters (above) prepared by two different groups and explanations (below) in these can be seen. From the poster presentations, it is understood that all of the groups are not at the same level in justifying equality of the probability in the draw. One of the groups (on the left) provided a correct explanation and stated that probability of the drawing does not change with respect to the order based on probability calculations. It is seen that the other group (on the right and below) provided an explanation that uses wrong probability calculations. Other than these, although some groups saw in their own calculations or in others' calculations that the probability was equal, they stated their opinions related to the possibility that the chance factor would change the probability. It can be stated that sense-making was not understood completely by these students. On the other hand, other students stated that the order was not important and presented the same probability value, which they calculated for different orders in the estate draw, as the explanation for this.

[6] Writings in the figure 6: "In the draw probabilities does not change. That is to say, when the probability of the first one's having a sea view apartment is 3/10, total probability of having sea view apartment is 3/10. The person drawing, shouldn't lose the hope. Even in the last drawing sea view apartment is possible. But the possibility of not getting the sea view is (always) higher. Drawing does not change the probability of getting sea view apartment." "In the mathematical calculation above, the probability of getting a sea view apartment is 1/5 for 3rd person, probability of not getting is 4/5. Even it seems very low at first sight, for each person probability of getting a sea view apartment is equal and low. Even for the 3rd one still there is possibility of getting. Even there are 2 sea view house, still there is possibility for 3rd one getting it."

In conclusion, in order to be able to perform the performance task including a real-life situation, the students tried to benefit from probability. It is considered that although it is important to complete the performance task by reaching other results, the more important thing is to create a perception in students in relation to the fact that they can benefit from probability in real-life processes.

3. Findings related to the Students' Attitudes toward the Subject of Probability

To answer the question of how the instruction designed via UbD affects students' attitude toward subject of probability, findings from attitude scale and individual interviews will be addressed. Firstly, the experimental and the control groups' group means in the pretests and the post-tests related to the variable of attitude are presented in table 12.

Table 12. The Experimental and the Control Groups' Attitude Score Means

	Pretest			Post-test		
	Number of Stu.	**Mean**	**Standard Deviation**	**Number of Stu.**	**Mean**	**Standard Deviation**
Experimental	27	107.9	25.4	27	140.9	18.0
Control	25	105.4	19.3	25	121.2	26.1

When table 12 is examined, it is observed that the experimental and the control groups' attitude score means increased in the post-test compared to the pretest. Moreover, while the groups' pretest score means are close to each other, there appeared a great difference between their post-test scores in favor of the experimental group. To determine if the difference between post-test attitude score means in favor of experimental group is significant or not, the results of covariance analysis in which pre-test attitude scores are presented in table 13.

Table 13: The ANCOVA Results of the Experimental and the Control Groups' Post-Test Attitude Score Means

Group	Test	Corrected Mean	Difference of Means	Standard Error	p-value
Experimental	Post-test	140.7	19.326	6.1	.003
Control	Post-test	121.3			

According to Table 13, the mean of the attitude scores which the experimental and control groups in the post-tests differed significantly in favor of the experimental group. Based on this finding, it can be stated that the procedure applied to the experimental group developed their attitude scores in a positive way.

In order to get a better explanation of this increase in attitude scores of the experimental group and to have information about students' feelings and behaviors during the lessons design via UbD, students' views about these issues are presented in table 14.

Table 14. Students' Views on Their Feelings and Behavior During the Lessons

Theme	Explana.	Sub-Theme	Code	Data Resource	Freque.
Feelings	*Students' feelings about and participation in the lessons*	Positive feelings	Enjoyable, amusing, pleasant	I: m4,f1,2,3,	4
			Not boring	I: f1,4,m5,6	4
			Feeling that s/he is getting attention	I: m6	1
		Negative feeling	Feeling bored when waiting for others	I: m3	1
Behaviors		Contribution to participation in lessons	Participation due to pleasurable activities	I: m2,4,f1	2
			Participation due to interest-arousing activities	I: m1	1
			Participation by overcoming the thought that they are difficult	I: f3	1
		No contribution to participation in lessons	Limited participation due to lack of background knowledge	I: m1	1
			Having difficulty	I: f4	1
			It did not affect, already participant	I: m6	1

When table 14 examined, the students' feelings were mostly positive during the lessons. The only negative opinion was a student's statement related to feeling bored of waiting for others during the lesson. The students stating positive opinions stated that they felt pleased in the lesson, the lessons were amusing and derived pleasure from the lessons. Moreover, the students especially emphasized that they did not get bored in the lessons.

> F1: "I derived pleasure; it was good, amusing; I did not get bored. To tell the truth, both I did not get bored and it made me participate in the lesson."

> M5: "I did not get bored of probability, for example, there happened nothing like getting bored; I felt more comfortable compared to other subjects."

> M4: "Sometimes, I exhibited behaviors spoiling the lesson, but most of the time I listened to the lesson, I mean, my teacher. What I felt was good; I mean, it was good, my teacher. I derived a lot of pleasure, I mean, it was good, my teacher. The lessons were enjoyable. We did our lessons joyfully"

One student put forward a rather interesting positive opinion and stated that the teacher showed interest and spend effort like in a private school.

> M6: "As if we were in a private school, you showed a lot of interest; you spent a lot of effort."

That the students derived pleasure and did not get bored in the lessons was rather positive and made us consider that a step was taken toward coping with an important problem especially in math lessons.

Related to the participation in lessons, some of the students stated that the design-based instruction increased their participation; less of them stated that it did not affect their participation. The students also stated that the lessons taught via activities were not boring, but even more enjoyable and amusing and this increased their participation. Moreover, it was a shared opinion that the teacher adopted a questioning approach and the materials were arousing interest.

M4: “Since it is enjoyable, everyone is participating in the lesson. Since you are learning everyone's idea, you are becoming more knowledgeable.”

M1: “Even if we do not participate in the lesson, you are continuously asking all of us questions, anyway. I mean, generally. I think it increased our participation. To tell the truth, you are wondering much more. For example, you are handing out worksheets and we are wondering much more and asking more questions.”

The students thinking that this way of teaching did not affect their participation very much generally attributed this to external causes. One of these was their not having a good background for the math course. Another was that since they were already participant students, it did not increase their participation. Only one student stated having difficulty in accompanying group works.

M1: “Frankly speaking, since I haven't got a good math background, I cannot do much”

M6: “Not, it does not make much difference; anyway I usually participate in lessons”

F4: “I mean, although it did not make much difference, I tried to participate; again I tried to understand by myself. I had some difficulty in collective works.”

As a result, it is possible to state that, according to the students' opinions, the instruction carried out via the design based on understanding increased the students' participation in lessons. It is considered that increase in participation corresponds to the designed instruction’s philosophical approach based on constructivism.

This scene emerging in students’ views about their feelings and behavior during the lessons seems compatible with the positive significant change in the attitude scores of the experimental group. Even, it is thought that students’ describing the lessons as enjoyable, amusing, pleasant and stating that the lesson increased their participation brought the rise in the attitude scores.

4. Findings related to the students’ opinions about an instruction designed via the UbD

Students’ opinions about the instruction designed via UbD were obtained from individual interviews. Findings coming from these interviews are presented table 15.

Table 15. Students’ Opinions about the Instruction

Theme	Explan.	Sub-Theme	Code		Data Resource	Frequ.
The way of teaching the course	*Students’ opinions about the designed instruction*	Positive opinions	Teaching via activities, using visuals		I:m2,3,4,6;f2,3,4	7
			Effectiveness of the worksheets		I: m1,5;f2,3	4
			Solving plenty of good examples		I: m3,4,5,6	4
			Discussion and exchange of ideas		I: m3,f1	2
			Positive opinions about the collaborative work	Benefiting from the activity-peer	I: f3,m1	2
				Seeing that there might be different ideas	I: f2,f3	2
		Negative opinion	Negative opinions about the collaborative work		I: m3,m5	2

As it is seen in Table 7, the students' general opinions about the way of teaching the course are positive. The negative opinions stated by two students resulted from their preferring individual work rather than group or pair work.

M3: “It is better to do it individually. Whatever I do, my partner does the same, too. For example, A.”

> M5: "If sheets were given one after another, it would be better. Instead of looking at one's sheet and telling another, everyone would do it by themselves; they would solve better. They would try to understand."

Despite this, some students found working together positive and stated benefiting from it. One of these students described the situation like this:

> F3:"We, two, are solving. I and my friend are not taking it in the same way. I'm wondering why my friend has done it in that way and I'm also wondering if I have done it in the wrong way; if so, then I'll do it in that way. We always did this together. When my friend did it in a certain way, I started to question myself why I had not done it in that way. I'm a person with a bit of attention deficit. But I'm asking my friend and s/he is telling me. I'm learning more things."

The opinions of the students stating positive opinions mostly focused on performing activities in lessons and the use of visual materials and elements.

> M2: "It was good; anyway, I understand through activities. I understand nothing on the board, but when there is an activity, I usually have a mental image of things. I mean, it was good."

> M3: "Better, my teacher. For when I write a lot, I mean when there are no visuals, I do not do a lot."

> F2: "I care visual things more.. Although I understand when I listen to something, but when visuals are added, I understand better; I mean, it is more effective for me."

Moreover, the students consider that solving plenty of good examples and using worksheets are effective. Finally, it is understood that the students found the discussions held in the lessons and the exchange of ideas done in this way positive.

In conclusion, it can be stated that the students found the instruction designed via the UbD positive in general. The students' expressing their thoughts by basing them on the details related to the course are making us consider that they are realistic in their opinions.

Discussion and Conclusion

The present study was conducted in order to reveal how the instruction designed via UbD affects students' level of achievement on probability, level of attitude toward probability, and sense makings related to probability and what the students' opinions about the instruction are, and following results are obtained. First of all, it was observed that the achievement scores of the experimental group were higher than those of the control group and, at the end of the instruction; the achievement score mean of the experimental group was higher than that of the control group. However, the difference between the groups' means was not statistically significant.

It can be stated that this is not in line with the studies reaching findings indicating that teaching probability using methods based on discovery and concrete materials (Gürbüz, 2006; Memnun, 2007), worksheets prepared with a constructivist approach (Işık & Özdemir, 2014) and collaborative methods (Keskin & Kılıç, 2016; Ünlü & Aydıntan, 2011), and enabling group work and group discussion in the classroom (Duran, Doruk, & Kaplan, 2017) will increase students' achievement levels. Considering the design of the instruction and the research set up, it should be stated that this is an expected result. Because, in accordance with the education system in Turkey, the academic achievement addressed in this study is related to the level of reaching the objectives of the probability subject in the curriculum and mostly based on remembering the probability concepts and making probability calculations correctly. However, the instruction designed in this study does not looking for academic achievement not in the first place, rather, it aims comprehending importance of probability in decision making process and using probability concepts and calculations in this process. It is thought that although using discovery learning and concrete materials are common aspects with those mentioned studies, main aim of the design based on UbD is different. Accordingly, more time and effort was allocated for activities including using probability in decision making and other real-life situations and expertise in a question type, solving it repeatedly, was never a primary objective. Even so, most of the students made calculations correctly in open-ended exams and performance task, the experimental group was not behind the control group in the achievement test. Furthermore, nearly all of the students from the experimental group stated that they have learned the subject of probability effectively and attribute this to the activities in the lessons and style of teaching. This finding is more than positive views about the teaching, it is thought students' self-efficacy levels related to learning probability could have increased and return back to students as motivation and self-confidence.

As learning probability, despite of students' reaching academic goals like remembering concepts and making calculations correctly, they have trouble in making sense of these. In the present study, examining understanding by design, the students' sense-makings related to the subject of probability showed themselves in the interviews, open-ended exams and performance tasks. The most important result is the students' having developed sense-makings related to the importance and use of probability in the decision-making process. In the interviews, all students stated the importance of probability in decision-making and an important number of them stated this opinion via different ideas. In relation to the use of probability, the students could give examples, mainly those which were given in the lessons. Furthermore, it was observed that, in the decision-making situations given in the open-ended exams, gradually more students started to use probability in their decision-making processes. When the contribution of probability to real-life decision-making situations (Halpern, 2003; Sharma, 2006) is taken into consideration, it can be stated that the students have been developing skills which they can use lifelong in addition to their academic lives. In the same way, the students gradually became better at explaining what the calculations which they made in the open-ended exams mean as well. Moreover, in the performance task at the end of the instruction, independent from the correctness of the calculations, all the groups had a tendency toward making sense of the calculations in the way they did and basing their decisions on these. It is considered that the appearance of the mentioned sense-makings was related to the inclusion of situations in which probability was used in the decision-making processes in the activities (e.g., daily decision with hourly rainfall report), the students' having encountered real-life decision-making situations in the evaluation processes (e.g., ordering a coffee, estate draw) and asking the students why they did them in that way during the instruction (e.g., supporting/refuting the fair attendant's claim). Starting from this result, it can be stated that the designed instruction will contribute to the solution of the problems which are claimed as important for teaching probability in some studies. These problems are inability to know what the calculations mean (Ben-Zvi, 2004; McGatha et al., 2002), inability to make sense the calculations correctly and make relationships between probity concepts and other concepts (Çakmak & Durmuş, 2015). This finding is congruent with results (Duran et al., 2017; Gürbüz, Erdem, & Fırat, 2016) claiming that in teaching probability, using activities including probability experiments with concrete materials and discussing the reasons behind the concepts and calculations used provide students to learn the probability concepts and calculations better and more meaningful.

On the other hand, mentioned studies (Duran et al., 2017; Gürbüz et al., 2016), similar to the most of other studies on teaching probability, are administered at the elementary level and probability topics they focus on are at more basic levels. As it is found in the present study either, more students have achievement on calculations and sense making for probability topic with lower level. However, in this study, the instruction designed via UbD includes higher level topics like conditional probability and some of the students become successful in terms of calculation and sense making even for this topic. In a study focusing on this higher level topics and using inquiry based learning, in spite of an improvement in students' inquiry skills, it is found that they made the calculation via the methods they had already know before the instruction (Chong, Chong, Shahrill, & Abdullah, 2017). At this point it can be said that the study on instruction designed via UbD has a positive effect on making students to reach objectives based on understanding in a higher level probability topic.

According to another finding, in the students' attitude toward probability scores of the students, the experimental group's mean increased more compared to the control group. More importantly, the experimental group's attitude score mean after the instruction is much higher than the control group's attitude mean score after the instruction and this is statistically significant. Therefore, it can be said that the instruction based on UbD increased the students' attitudes toward probability. In this study, the students' attitudes toward the subject of probability is in line with other studies (Arı & Topçu, 2013; Bulut, Yetkin, & Kazak, 2002). In the pretest the mean was at the "I might agree" level of the six-point Likert. However, in the post-test, while this level increased to the "I agree" for the experimental group, it remained at the same level for the control group. To understand this positive attitude change toward probability better, students' feelings and behavior can be considered. According to the findings obtained from the interviews, the students described the designed instruction as composed of enjoyable, not boring and fun lessons and stated that the teacher increased their participation in the lessons by arousing interest. It is considered that the inclusion of interesting activities (e.g., "Monty Hall Problem"), the visual contents related to the subject (e.g., the video of "By Any Chance), the experiments made via concrete materials (e.g., drawing a color ball) and group works in the lessons had an effect on the students' opinions and attitude levels. There are both international (Green, 1982, Sharma, 2006, Watson & Kelly, 2007) and national (Çakmak & Durmuş, 2015; Gürbüz, Toprak, Yapıcı, & Doğan, 2011; Kutluca & Baki, 2009) studies found that the subject of probability described by students as a fearsome and difficult subject. For this reason, in the present study, that the students developed positive attitudes and feelings toward probability can be considered as a noticeable result.

Finally, it can be stated that the students' opinions about the probability instruction designed via the UbD were generally positive. The students explained their positive opinions about the lessons by expressing their satisfaction from performing activities, using worksheets, making discussion, solving good examples and doing collaborative work. In addition to this, in the individual interviews students defined the design-based probability instruction as enjoyable, amusing, not boring and interest-arousing. According to students, this new situation increased their participation to the lesson and helped them to understand probability topics better. These results, from the individual interviews, seem to be in accordance with the quantitative finding of the increase in the students' attitude scores toward the lesson. It is understood that factors like interesting materials, concrete experiences and real-life situations contributed to students' development of positive opinions. In teaching probability, Duran, Özdemir and Kaplan (2015) reached similar student opinions by using similar activities with problem based learning approach.

Sharma's (2015) claims that an instruction taking social constructivism and students' foreknowledge into consideration can contribute to the development of probabilistic thinking. It can be stated that results of this study support that claim. The general idea reached is that with the help of an instruction based on authentic processes that include using probability in real-life situations rather than memorizing definitions in subject of probability and difficult calculation procedures, it is possible that students can make sense of probabilistic calculations, make judgements by using these and support these judgements.

Suggestions

In this study an instructional design based on understanding was used in teaching the subject of probability, whose concepts and calculation challenge students to make sense. First of all it is thought that in teaching the subject of probability and other subjects, in which students having difficulty on making sense of mathematics done, enabling and using instructional designs are important. In this context some suggestions for the practitioners are presented below:

- In order to cope with problem of inability to make sense of mathematical concepts and calculations, in both instruction and evaluation process students' experiencing the situations which require using these concepts and calculations. Especially for the subject of probability, probability experiments with concrete materials and corresponding mathematics should go hand in hand.
- In the design, importance of the subject of probability in terms of decision making was taken as the big idea and the instruction is constructed among this idea. Using big idea is suggested because it helps both the designer does not get lost during the designing process and the student reach an important and life related consequence. For the subject of probability, the big idea can guide both student and designer about the aim for which all the means are.
- In teaching probability, evaluation is generally related to making calculations correctly and this causes procedure (or mean) become the main aim. In this study, it is observed that the students who could not make the calculation completely, make senses of probability. For this reason, by the designer, what is important should be determined more clearly and in accordance with it evaluation tools should be produced. Justification of the mathematical calculations should be permanent part of instruction for both the subject of probability and other mathematical subjects.
- In the designed instruction, in all of the activities except for individual exams, collaborative work was included. Especially, pair work including students with different success levels were used effectively. For the subjects requiring higher level understanding like probability, pair work should be used with precaution (i.e. using different colored pencils) ensuring that both of the students' make effort.
- Performance task is suggested for almost all instructional areas because it requires both using the acquired knowledge and skills for a more general aim and revealing her/his potential skills of communication, collaboration and creativity.

Consequently, to get more general results for effectiveness of using UbD in teaching probability, examining effect of it in different contexts is suggested.

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Appendix 1. Work Sheet Example

Following is the work sheet for second two hour lesson. It is compressed and attached.

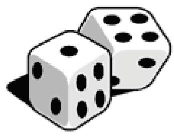
1st Activity

The activity in the lesson plan presented in the text.

- How can you calculate you probability of winning if there is a car behind one of the three doors?
- When the showman open one of the doors you did not select, do change the door you selected?
  - I change because ....
  - I don't change beacuse ....

Examples

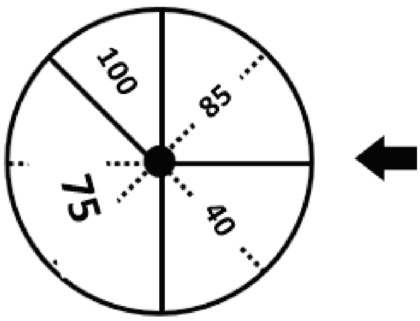
Find the probability of a coin's come up two consecutive tails in a flip the coin game.



Find the probability of the numbers of dices rolled together comes up are the same.

2nd Activity and Examples

The wheel below has eight equal slices and can be spun.



A teacher lost the exam papers, will give students' exam scores by spinning the Wheel.

- Which score, do you expect, is the most frequently given? Why?
- Which score, do you expect, is <u>not much</u> frequently given? Why?

- Calculate probability getting score of 100. ....
- Calculate probability getting score of 85. ....
- Calculate probability getting score of 75. ....
- Calculate probability of fail (getting 40)? ....
- Calculate probability of pass (not getting 40)? ....

What kind of relationship does exist between probabilities of pass and fail?

- In a box, there are 3 blue, 5 yellow and 4 red balls. For a ball drawn from this box, calculate
  - The probability of being blue and not being blue
  - The probability of being yellow and not being yellow.
  - The probability of being red and not being red.

Appendix 2. Open Ended Exams

Open-Ended Exam 1: Simple Probability

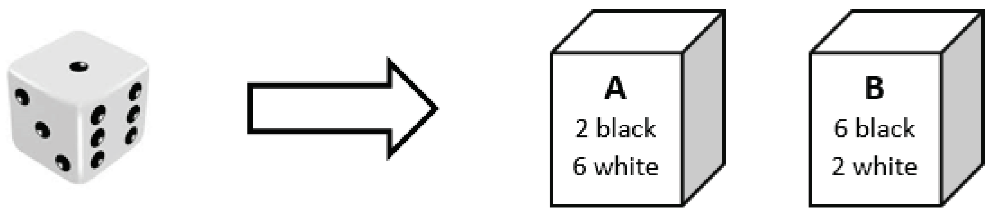
Café El Escorial		
CAFÉ	*Solo*	*Caramelo*
Cubano	2	3
Décafe	3	2,5
Espresso	1,5	2
Griego café	3,5	2
Havana Colada	2,5	3
Leche	2	2,5

You are in Havana in a coffee house and the only menu is above. In the menu, there are 6 different menu and two different aroma for each coffee. Because you can speak Spanish, you will make an order showing on the menu.

- Write a set consisting of all possible orders.
  - What is the name of this set?
- Assume that there are 5 coffee you will like
  - What is the probability of ordering a coffee you will like?
  - What is the probability of ordering a coffee you will not like?
- Is it wise to order a coffee? Why?

Open-Ended Exam 2: Conditional Probability

In a fair you are playing a chance game. You are supposed to roll a dice and you draw a ball from a particular box depending on what dice comes up. Draw a black ball and win the game.



A ball is drawn from box A if the dice comes up 3 or bigger, from box B if it comes up smaller than 3. In box A, there are 2 black – 6 white balls, in box B there are 6 black and 2 white balls.

- What is the probability of winning the game?
- What is the probability of losing the game?

Fair staff defends that total number of black and White ball in the bosex are equal and so the probability of winning and losing.

- How do you support or reject the staff's claim?

Appendix 3. Implementation Form of the Performance Task

Think that you are a building contractor and raised a building having 10 flats. 3 of these flats have sea view and the other 7 flats have not. All of the 10 customers want a sea view apartment accepting paying more.

Then, a draw had been set up. Door numbers were written on cards and put in a box. One by one each customer will draw a card and get the apartment written. Of course the card won't be put into the box again.

Customers insists on drawing first to get the one of those 3 apartments. You want to show them the order of the drawing does not affect their chances.

- You are expected to prepare a poster.
- In this poster, you are suggested to present evidences for both
  - The customer have some probability knowledge and
  - The customers have not

- Your performance will be evaluated by the mathematics teachers
- Standards of this evaluation are
  - Using probability knowledge and calculations efficiently
  - Making calculations concrete by visualization.