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The Effects of Using Geometry Activities Based on Dienes' Principles on 4th Graders' Success and Retention of Learning *

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Abstract

This study aimed to investigate the effects of geometry instruction based on Dienes' principles (*constructivity, dynamic, mathematical variability and conceptual variability principles*) on 4th graders' geometry success and retention of learning. It was a pretest-posttest control group quasi-experimental study. The study group comprised three classes of 4th graders from three different elementary schools in Nevşehir who had been shown to be identical via a "Geometry Level Identification Test". The study was conducted with two experimental groups and a single control group. The study took 39 class hours. While instruction in the experimental groups was based on Dienes' principles, the researcher did not intervene in the instructional process in the control group. The data collection tool used in the study was the "Geometry Level Identification Test". In comparing the pretest, posttest and retention test mean scores, Covariance (ANCOVA) and paired-sample t-test analyses were used. The results revealed that the experimental groups, which experienced learning activities based on Dienes' principles, had better geometry success than the control group where instruction was not manipulated. While no significant difference emerged between the "Geometry Level Identification" posttest mean scores of the two experimental groups, both groups differed significantly from the control group. The retention test given three weeks following the study showed that some information had been forgotten by all three groups. The retention test scores of the groups were 5-7 points lower than their posttest scores. Even though a significant difference was not found between the retention scores of the groups, it may be stated that the decline in the control group was noteworthy.

Keywords

Dienes' principles
Geometry
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Introduction

The foundations of geometry, an important sub-branch of mathematics, are laid in the preschool period and it has an important place in the school curriculum. However, the abstraction that is present in the nature of mathematics and geometry make it difficult for children to make meaning of geometry. Making meaning of such abstraction is possible by the right kind of education and rich experiences in the instructional environment. A geometry instruction devoid of rich experiences limits children's understanding (Clements, 1998; Faggiano, 2012).

It may be said that success in geometry education is lower than the other branches of mathematics in Turkey. Geometry is one of the areas in which Turkish children score lowest in many international exams (See Programme for International Student Assessment [PISA], 2003, 2006; Trends in International Mathematics and Science Study [TIMSS], 1999, 2011). Similarly, Turkish students also perform poorly in the geometry section of national exams. In the 2012 Transition to Higher Education Exam (YGS), students obtained a mean score of 6.73 in the geometry section which included 30 items, and in 2013 they obtained a mean score of 4.15. Later, in 2015, the geometry mean score of 757.768 candidates was 3.78 (Student Selection and Placement Center [ÖSYM], 2012, 2013, 2015).

Due to such problems students have with geometry, both national and international literature focuses on how to help students improve their understanding of this branch of mathematics. Research has been conducted into the use of various methods, techniques and strategies in the teaching and learning of geometry. The present study focuses on the effects of using the alternative mathematics education approach of Dienes' principles on student success in geometry. Prior to explaining Dienes' principles, it would be useful to discuss the process of geometry instruction.

The Process of Geometry Instruction

Geometry is not only a discipline in which theoretical thought is constructed, but it is also an indispensable part of our cultural experiences which are vital in many aspects of our life (Faggiano, 2012). Geometry also gives us powerful tools to represent and solve problems in different areas of mathematics, other school courses and daily life (National Council of Teachers of Mathematics [NCTM], 2001, pp. 1-2). Therefore, grasping geometry is an important mathematical skill in all stages of instruction.

Although researchers emphasize that geometry is important for students in all stages of education (Clements & Battista, 1992; Clements, 1998), students' geometry performance does not reflect its significance (Burns, 2007; Clements & Battista, 1992). National (YGS, Transition from Basic to Secondary Education [TEOG]) and international exams (TIMSS, PISA) show that geometry is the field in which students are minimally competent. This invites the question "Why all these difficulties?". Researchers list the following as the reasons for the difficulties experienced in geometry:

- Instructional approaches selected to teach geometry,
- Teacher failure to go beyond typical curriculum materials,
- Insufficient teacher knowledge of geometry,
- Insufficient examples of geometry concepts in mathematics textbooks,
- Students solving geometry questions with memorized knowledge rather than deep understanding,
- Students not having a desired geometric thinking level (Fidan & Türnüklü, 2010; Gökbulut, 2010; Gökbulut & Ubuz, 2013; İnan & Doğan Temur, 2010; Olkun, 2005; Olkun & Aydoğdu, 2003; Toptaş, 2007; Yenilmez & Yaşa, 2008).

According to the international literature, the biggest reason for the difficulties students face in geometry include little emphasis on developing geometric skills, focusing on memorizing general procedures (i.e. multiplying two sides of a rectangle gives us its area or the sum of a triangle's inner angles is 180°) rather than understanding in the teaching of geometric concepts, insufficient education, teacher emphasis on other learning areas of mathematics such as arithmetic or measurement, linguistic difficulties in self-expression, developmental incompetencies in drawing, spatial presentation of mathematical ideas, the differences between the daily use of certain words and their use in mathematics (Burns, 2007; Dickson, Brown, & Gibson, 1984; Driskell, 2004; Heddens & Speer, 1995; Miller & Mercer, 1997).

It would be right to say that both national and international literature show most geometry-related problems to stem from instructional approaches. However, research places children's ways of learning geometric thinking in a structuralist position. Children construct geometric concepts in a gradual way from a perceptual to conceptual plane (Clements & Battista, 1992). The learning of geometric concepts by students is directly related to an individual's development and thinking levels (Toptaş, 2010). Environments which provide students with rich experiences are essential for the objective of the entire cognitive structure related to geometric concepts. Without such learning environments, individuals will not be able to make meaning of geometric concepts and learning does not go beyond mere memorization (Duatepe Paksu, İymen, & Pakmak, 2013, pp. 164).

For students to be able to understand geometric shapes, they need to discover the parts and qualities of these shapes. Only seeing pictures of geometric shapes and naming them is not enough to construct geometric concepts (Burns, 2007; Clements, 1998). For the teaching of geometric concepts, it is essential to include exploratory examples, relevant and irrelevant examples, reverse examples, best examples and different representations in classes (Clements, 1998; Cross, Woods, & Schweingruber, 2009). At the same time, geometry instruction needs to go beyond typical instructional materials (textbooks, exercise books, etc.) to enrich children's learning. Children need materials that describe their own experiences. Particularly for elementary school children, geometric shapes and materials should be used in classes as concrete learning tools (Clements, 1998). These support children's development in areas such as problem solving, communication and assumption, which are defined as essential mathematical skills by the NCTM and the Ministry of National Education. Learning by doing, touching and moving also helps develop a sense of geometric concepts. It is a widely accepted fact that environments where children can construct their own mathematical concepts contribute to the development of geometric thinking.

Dienes' Theory and Principles

Dienes considers the learning of the concept of mathematics as a process which covers abstraction, generalization and transfer (Dienes, 1960, pp. 18). Although he seems to have largely adopted Piaget's views, Dienes contributed significantly to cognitive psychological views on learning mathematics (Olkun & Toluk Uçar, 2012; Post & Reys, 1979). Dienes' main concern was the early learning of mathematical concepts. He argued that the teaching of these concepts should make more use of manipulative materials and play (Wisthoff cited in Gningue, 2006, pp. 41). In his own mathematics education theory, Dienes focused on the use of discovery type activities and student-centered manipulative materials (Fossa, 2003). He experimented with a group of constructed materials and postulated a series of principles for the use of these tools (Dienes, 1960; Dienes & Golding, 1971). Known by the name of "Dienes' principles", these include "*constructivity, dynamism, perceptual variability and mathematical variability*[1]" (Dienes, 1960). Dienes integrated mathematical structure with these principles that he developed. The four principles that are based on embodiment are explained below:

1- Constructivity Principle: This principle echoes the belief that children should be allowed to develop their concepts intuitively by starting from their own experiences (Cathcart, Pothier, Vance, & Bezuk, 2003; Olkun & Toluk Uçar, 2012; Post, 1981). In order for learning to take place in line with the

[1] This ordering is in line with Dienes' book (1960) *Building up Mathematics*.

constructivity principle, the use of concrete materials is vital. Constructivity principle is based entirely on the constructivist approach. Studies on this approach show that it increases students' mathematical success, gives them higher order thinking skills, and positively influences their interest and attitudes for the mathematics course (See Arseven, 2010; Ayaz & Şekerci, 2015; Çağlar, 2010; Duatepe Paksu & Ubuz, 2009; Işık & Çağdaşer, 2009).

2- Dynamic Principle: Dienes argues that all abstraction and thus all mathematics comes from experience and that concept development is based on a psycho-dynamic process. He states that experiences and learning should be planned within a process of consecutive cycles (Dienes, 1960). This process has been dubbed the "dynamic principle" by Dienes himself (Dienes, 1960; Dienes & Golding, 1971). In his dynamic principle, Dienes views the true understanding of a new concept as an evolutionary process which temporarily involves the student in three stages of activities: preliminary, structured and practice/reflective (Olkun & Toluk Uçar, 2012). The assumption is that, as long as activities in each stage are covered at the right time, mathematical concepts will be constructed gradually with their help (Dienes, 1960). The dynamic principle involves play-based instruction. Play-based or play-supported mathematics education studies have shown that play affects students' success, interest levels and attitudes (Altunay, 2004; Yiğit, 2007; Yücel Yumuşak, 2014).

3- Mathematical Variability Principle: When the related variables in a concept are held constant and unrelated ones are systematically changed, this concept may be perceived under different conditions and the generalization of the mathematical concept is reinforced (Olkun & Toluk Uçar, 2012; Post, 1981). In the mathematical variability principle process, concepts with variables should be learned via experiences that include the largest number of variables possible. The mathematical variability principle needs to be considered in the development of van Hiele geometric thinking (Hoffer, 1983).

4- Perceptual Variability Principle: This principle involves variables having a large influence area in concept development and offering children various perceptual aspects of the same conceptual structure in the learning environment so that they can understand abstraction (Dienes, 1960, 1964; Olkun & Toluk Uçar, 2012). Similar to the previous principle, perceptual variability also contributes to the development of van Hiele geometric thinking (Hoffer, 1983).

The common point of the four principles explained above (*constructivity, dynamic, mathematical variability and perceptual variability principles*) is that they show the importance of direct interaction with the environment in mathematics education. Dienes continuously emphasized that mathematics education is not a passive job, but asks for active physical and mental participation from students (Olkun & Toluk Uçar, 2012; Post & Reys, 1979; Post, 1981). Dienes also stated that mathematical concept development occurred in children with a psychodynamic process and that students' learning experiences should be organized according to these principles and levels (Dienes, 1960).

An Overview of Studies on the Variables of Geometry and Dienes' Principles

The literature reviewed for the present study shows that both national and international studies have been conducted into the teaching of geometry topics and the use of Dienes' principles in the instructional process. Studies at the elementary school level include the use of different approaches in geometry education (Efendioğlu, 2006; Olkun, 2003; Olkun, Altun, & Smith, 2005; Olkun & Sinoplu, 2008; Terzi, 2010; Tutak, 2008; Tutak, Türkdoğan, & Birgin, 2009). Different methods, techniques and strategies have been used in geometry education. Some researchers (Siew, Chong, & Abdullah, 2013; Siew & Chong, 2014) have based their studies on van Hiele's theory that geometric thinking is formed by passing through certain stages (van Hiele, 1959). Their studies on elementary school geometry education (grades 1 through 4) focused on van Hiele's geometric thinking levels and instructional stages. These studies have shown that instruction designed in line with van Hiele's levels and instructional stages have an important effect on student success (See Siew et al., 2013; Siew & Chong, 2014).

Similarly, many researchers (Gecu & Satici, 2012; Meng & Sam, 2013; Özçakır Sümen, 2013; Zaranis, 2014) studied the effects of technology use on geometry success as it gives students many opportunities to learn in mathematics education (Albaladejo, Garcia, & Codina, 2015; Battista, 2002; Olkun & Altun, 2003). Their studies have confirmed that technology-supported geometry instruction at elementary school increases student success (See Efendioğlu, 2006; Gecu & Satici, 2012; Meng & Sam, 2013; Olkun & Sinoplu, 2008; Özçakır Sümen, 2013; Tutak, 2008; Tutak et al., 2009; Zaranis, 2014).

Certain other researchers (Faggiano, 2012; Sarı & Bulut, 2013; Tutak, 2008) chose to investigate the effects of concrete material use on geometry success (Dienes, 1960). Studies conducted with 1st-4th graders revealed that concrete material use positively affects students' geometry success (See Faggiano, 2012; Martin, Lukong, & Reaves, 2007; Sarı & Bulut, 2013; Tutak, 2008).

In studies that made direct use of Dienes' principles (Gningue, 2000, 2006; Sriraman & English, 2005; Velo, 2001; Zhang, 2012), these principles were shown to enable the students to construct the abstractions and generalizations at the heart of mathematical ideas. The literature reports that Dienes' principles were found to contribute to students' mathematics success, to allow them to construct the abstractions and generalizations at the heart of mathematical ideas, provide students with novel experiences in concept development, and make meaning of the abstraction which is present in the nature of mathematics by perceiving the instructional process as a game (Gningue, 2000, 2006; Sriraman & English, 2005; Velo, 2001; Zhang, 2012).

Both national and international studies on geometry and the use of Dienes' principles in the instructional process have certain limitations. Particularly in Turkey, the practices employed in geometry studies at elementary school level (technology use, instructional approaches, etc.) were found to help students learn. However, the fact that geometry still emerges in many international exams (TIMSS, PISA) as one of Turkish students' weaknesses (Ministry of National Education [MEB], 2014, pp. 25) suggests that experiments do not last and they cannot be reflected in overall practices.

By the same token, studies using Dienes' principles were mainly conducted with more advanced grades (secondary school and above) and had positive effects on learning. Other observations were that most studies did not cover all of Dienes' principles (constructivity, perceptual variability, mathematical variability and dynamic principles) and that they were conducted overseas. Mathematics education studies in Turkey largely focus on general instructional methods (discovery learning, problem-based learning, drama-based learning, etc.) (Çilingir, 2015; Duatepe Paksu & Ubuz, 2009; Öksüz & Uça, 2011; Yücel, 2009), do not seem to be fully familiar with Dienes or his principles, and do not include any study dealing with principles developed solely for teaching mathematics, thus creating the need for the present study. Also, it is evident that Dienes' stages of the mathematics learning process were treated separately in many studies (Arseven, 2010; Ayaz & Şekerci, 2015; Çağlar, 2010; Çilingir, 2015; Işık & Çağdaşer, 2009; Öksüz & Uça, 2011; Yücel, 2009). It was believed that geometry education would benefit from considering the entirety of the principles of Dienes, who was a mathematician himself, studied mathematics education directly and believed in constructivism and active learning (Olkun & Toluk Uçar, 2012).

Another difference between this study and others is that the separately treated mathematics education strategies and methods of previous studies (learning through games, discovery learning, van Hiele thinking levels, etc.) are treated together in this study based on the constructivist approach. It is hoped that the 4th grade geometry and measurement lesson plans designed for this study will constitute an example for the learning situations created by the constructivist approach used in the 2015 Elementary Mathematics Curricula, become a resource for teacher candidates, and contribute to the literature. The present study focuses on examining the effects of 4th grade geometry instruction in line with Dienes' principles on student learning and its retention. To this end, answers were sought to the following problems and sub-problems:

Problem: What are the effects of geometry instruction on 4th graders' geometry success and retention levels in Experimental Groups 1 and 2, which offer instruction based on Dienes' principles, and in the Control Group, which the researcher does not intervene with?

Sub-problems:

1) *Is there a significant difference:*

a) *between Experimental 1, Experimental 2 and Control groups considering their "Geometry Level Identification Test" posttest mean scores corrected according to the pretest scores?*

b) *within Experimental 1, Experimental 2 and Control groups considering their "Geometry Level Identification Test" (pretest-posttest) mean scores?*

2) *Is there a significant difference;*

a) *between Experimental 1, Experimental 2 and Control groups considering their "Geometry Level Identification Test" retention test mean scores corrected according to the posttest scores?*

b) *within Experimental 1, Experimental 2 and Control groups considering their "Geometry Level Identification Test" (posttest-retention test) mean scores?*

The study is limited to the 4th grade geometry and measurement (area and circumference) objectives listed in the Elementary Mathematics Program (2009) which was in effect during the 2014-2015 school year as well as the year when the study was conducted.

Method

Study Design

This study was designed as a pretest-posttest control group quasi-experimental study (Büyüköztürk, 2014). Quasi-experimental models are preferred when the controls necessitated by true experimental models cannot be achieved or are not adequate (Karasar, 2012, pp. 99). In this design, effort is made to match participants from spontaneously formed groups (Büyüköztürk, Çakmak, Akgün, Karadeniz, & Demirel, 2009, pp. 206). The study used the quasi-experimental design as participants did not have the chance to be assigned randomly to the experimental and control groups. Even though the pretest-posttest control group design is a powerful research model, it also harbors a set of weaknesses such as the risk of a reduction in participant sensitivity due to the repeated use of measurement tools in the groups. That is why a certain time lapse is recommended between the completion of the experiment and the follow-up study (Heppner, Kivlighan, & Wampold, 1999). The present study conducted a retention test three weeks after the experimental implementation to check whether the effects of the instruction were still continuing.

Study Group

The study group included students from selected 4th grades of three state elementary schools at middle socio-economic level which were located in Nevşehir. The study group was chosen with the group matching method. This method involves choosing groups that are equal and/or similar to each other regarding the study variables (Eckhardt & Ermann cited in Büyüköztürk, 2014, pp. 22). In this study, 4th graders from three different schools took the "Geometry Level Identification Test" so that groups could be matched. The means and standard deviation values of the scores that groups obtained from this pretest are shown in Table 1.

Table 1. Means and Standard Deviation Values of Scores Obtained by Experimental 1, Experimental 2 and Control Groups in the Geometry Level Identification Pretest

Düzey Belirleme Testi*	**N**	$\bar{x}$	**Ss**
Groups	31	28.55	14.23
Experimental 1 Experimental 2	29	33.31	13.35
Control	25	31.72	13.46

* The highest possible score from the test is 76.

As shown in Table 1, the geometry level identification test mean scores of Experimental Groups 1 and 2 and the Control Group were $\bar{x}$ = 28.55, $\bar{x}$ = 33.31 and $\bar{x}$ = 31.72, respectively. The mean scores of the groups were compared by using *One Way Analysis of Variance* (ANOVA) (Büyüköztürk, 2010, pp. 48-54; Can, 2014, pp. 147-158). ANOVA results are given in the findings and interpretations section of the study.

The study used two different experimental groups in order to explore any differences created by the class teacher or the researcher. In other words, the aim was to reveal the effects, if any, of the class teacher or the researcher on the instructional process. It is important to establish whether the teacher or the researcher may be a variable influencing the experimental study, and whether the results depend on individuals.

The aim in choosing 4th graders as the study group was to reveal their state as they finish elementary school as mathematics is a field in which prerequisite relationships were strong. This grade was chosen as success during the last year of elementary school predicts success in future grade levels (Bloom, 2012, pp. 38-43) and as 4th grade geometry objectives cover early learning as well. In addition, students graduate from the first stage of basic education at the end of 4th grade and their geometry objectives achievement levels at this point constitute a topic of interest. ıt is also worth noting that the objectives investigated in the present study were largely those that 4th graders were acquainted with for the first time.

The schools were chosen from the middle socio-economic level because of a reluctance to include in the study extreme examples which might bring in very positive or negative factors (at least with regard to resource factors). Also, the aim was to examine and interpret data from ordinary/mainstream schools.

Data Collection Tools

While collecting data for the present study, both the experimental and control groups were given the 76-item "Geometry Level Identification Test" developed by the researcher as pretest, posttest and retention test. This 4th grade level identification test was based on the objectives listed in the Geometry (Angles and Angle Measurements, Triangles, Squares and Rectangles, Pattern and Decoration, Symmetry and Geometric Objects Sublearning Domains) and Measurement (Circumference and Area Sublearning Domains) domains of the "Elementary 1-5 Mathematics Program" (MEB, 2009). This study is limited to the learning domains and objectives included in 2015 Elementary Mathematics Program (grades 1-5). After identifying the objectives in the learning domains, 3 to 5 multiple choice questions were written to measure each objective. Some of the questions were adapted from other sources (Morpakampus.com; Mutlu Yayıncılık, 2014; Okulistik.com) and others were written by the researcher.

As the questions were written, tables of signs were prepared for content validity. In these tables of signs, multiple choice questions were prepared for the 24 objectives related to the "Geometry (Angles and Angle Measurement, Triangles, Squares and Rectangles, Patterns and Decoration, Symmetry and Geometric Objects Sublearning Domains) and Measurement (Circumference and Area Sublearning Domains) Learning Domains and Sublearning Domains". The total number of questions was 103. In order to check whether the questions had content validity, they were sent for the review of class teachers who have taught or are teaching fourth grade (3 teachers), mathematics educators (1 professor, 1 assistant professor and 1 research assistant), a program development expert (1 associate professor) and a measurement and evaluation expert (1 assistant professor). Assessments were made with the criteria given in Table 2. In addition, the intelligibility and feasibility of the questions were tested through a pilot trial with two fifth graders.

Table 2. Evaluation Criteria for the Questions in the Pilot Trial

A sample question from the pilot trial:

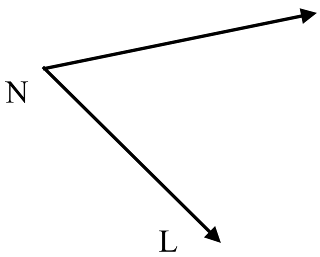
Learning Domain: Geometry

Sublearning Domain: Angle and Angle Measurement

Objective: The student names the angle and indicates it with its symbol.

In math class, Emel draws an angle and names it. The she uses symbols to represent the same angle that she drew in a different way. Which of the below representations is wrong?

A) ∠NİL B) ∠N C) ∠İNL D) ∠LNİ



Valid Question	Invalid Question Must Be Corrected	Must Be Corrected	Your Suggestion

In order to conduct item analysis on the level identification test questions revised upon expert feedback, they were implemented on a total of 261 students in two groups: the group with no information (4th grades) and the group with prior information (5th grades). The scores obtained from this implementation were used to calculate item difficulty index (p), item discrimination index (rjx) and reliability coefficient (KR-20). Items with an item difficulty level outside .40–.70 were excluded from the test (Turgut, 1990, pp. 267), and those with a discrimination index of .30 and above were included (Tekin, 1991, pp. 249). The general difficulty value of the test was (p)=.55 and its KR-20 reliability coefficient was .912. As a result, the 76-item level identification test for elementary students was obtained. Thinking that it would be difficult to take this test in one sitting, it was divided into sublearning domains (angles and angle measurement, squares and rectangles, area and circumference) and given in three sessions (25, 26 and 25 items). The students were given one class hour for each test.

The Instructional Process

The main implementation for both experimental groups and the control group took 39 class hours (approximately 10 weeks). In all three groups, the implementation was complete within the 10th week. After a three-week gap, the groups were given a retention test. Mathematics classes in the experimental and control groups were four hours weekly. The decision to allocate groups as experimental or control was made on the basis of convenience of study and easy dialogue with the teacher. Information about the process in the experimental and control groups is given below under the subtitles "Studies in the Experimental Groups" and "Studies in the Control Group".

Studies in the Experimental Groups:

In Experimental Groups 1 and 2, the instruction was based on Dienes' principles. In Experimental Group 1, the instructional activities based on Dienes' principles were implemented by the class teacher. Prior to the experiment, the class teacher was given information by the researcher about the processes to be experienced. First, the teacher was informed about Dienes' principles and the processes behind these principles were explained practically. Sample course plans and instructional tools were described to the teacher. The teacher in Experimental Group 1 received training twice weekly for three weeks. The tools and materials necessary for the lessons were supplied by the researcher. The researcher gave the tools and materials and information about them to the class teacher.

In Experimental Group 2, the researcher implemented the instructional process based on Dienes' principles. Prior to the experiment, students spent four class hours with the researcher in order to become familiarized.

Experimental Groups 1 and 2 both followed the lesson plans developed by the researcher in line with Dienes' principles. As these lesson plans were being developed, theories, articles and theses on Dienes' principles were examined (Dienes, 1960; Dienes & Golding, 1971; Olkun & Toluk Uçar, 2012; Post, 1981; Sriraman, 2008; Sriraman & English, 2005). Following this, field experts were consulted (1 professor, 1 associate professor, 1 assistant professor) and the "Dienes' Principles Criteria Set" was developed by the researcher in order to effectively design lesson plans to be used in the instructional process. This criteria set covers Dienes' principles (dynamic principle, perceptual variability principle, mathematical variability principle and constructivity principle) and is presented in Table 3.

Table 3. Dienes' Principles Criteria Set

Critical Behaviors Regarding the Dynamic Principle:	**Critical Behaviors Regarding the Perceptual Variability Principle:**	**Critical Behaviors Regarding the Mathematical Variability Principle:**	**Critical Behaviors Regarding the Constructivity Principle:**
A- In play activities: • Children get ready for class. • Give their interest. • Come in contact with real life. B- In semi-structured activities: • Children can use previous knowledge. • Reveal relationships. • Reveal patterns. C- In structured activities: • Children reach concepts. • Reach rules. • Reach formula. • Reach definitions.	• Children understand that even though a concept/shape is in a different direction or position, it still has the same qualities. • Recognizes perceptual equals of the same concept (representing the concept with different concrete materials).	• Children understand that when the variables related to a concept/shape are held constant, systematically changing irrelevant variables will not change the qualities of the concept/shape. • Grasps a concept through an experience including the highest number of variables.	• Children construct knowledge and reach the concept/rule/formula/analysis by themselves. • Transfers learned information to new situations.

Within the frame of the specified criteria set (Table 3), lesson plans were prepared for the 24 objectives in the MEB (2009) mathematics program. As lesson plans were being prepared, interrelated objectives were linked to *transfer activities*. In Experimental Groups 1 and 2, symbols were developed to remind the class teacher and the researcher of all of Dienes' principles. The symbols were inserted in the relevant places in the lesson plans. These symbols designed in line with the principles are shown in Figure 1.

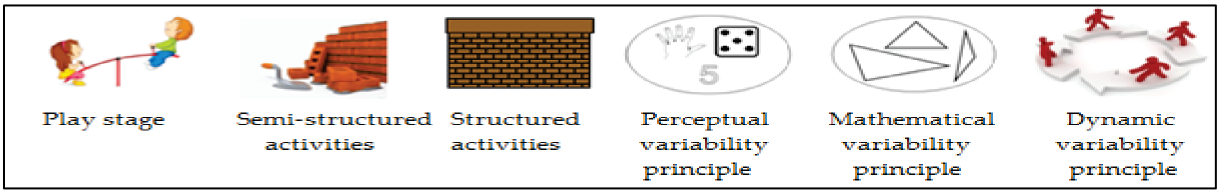


Figure 1. Symbols Developed for Dienes' Principles

The criteria set (Table 3) was used to prepare the following sample lesson plan to be used in the experimental groups:

In the first of the three stages of the dynamic principle, the play stage, students were made to play tetris over the internet, referring to the circumference sublearning domain. One of the members in all groups were allowed to play this game. Later students were told that they could also make tetris pieces themselves. Each child was distributed unit squares to make 4 (tetromino) tetris pieces. They were asked to form different tetrominos by using the squares they were given.

In another dimension of the dynamic principle, semi-structured activities, children made various animal figures, houses, etc. by using the 4 (tetromino) tetris pieces they used in the free play stage. Then, they were requested to form planar shapes with their 4 tetris pieces. They were asked whether the shapes they made had circumference length and their answers were elicited. They were then asked: “How about calculating the circumference length of planar shapes?” After having been told that 5 (pentomino) tetris pieces were also possible, they were distributed 5 tetris pieces by the teachers. The children were asked to use these pieces to form those planar shapes that they knew.

In the final dimension of the dynamic principle, concept reaching, children were requested to draw planar shapes with 5 (pentomino) tetris pieces on plotting paper and were asked how they would calculate circumference length. Then they were told to find how many unit sides were present in the circumference of the pieces they drew. In this process, they were guided to reach the idea (concept) that circumference length is the sum of side lengths of the shapes.

Within the scope of the mathematical variability principle, students were asked to first use different materials to form planar shapes with the same and different circumference lengths (geometry board, tetris pieces, etc.) and then change their direction and position. Circumference measurements were then repeated with the new direction and position. In this way, students were led to understand that changed direction and position do not change circumference length.

Within the scope of the perceptual variability principle, children were distributed 5 tetris parts, geometry board, toothpicks and matchsticks so they could make planar shapes. The children were asked to make different planar shapes by using these materials. With the activities, children were enabled to reach mathematical concepts by seeing perceptual equals (with different materials) of the same conceptual structure (planar shape).

An effort was made to ensure the following in the worksheets, which were designed by considering the critical behaviors in Dienes criteria set and the principles of constructivist learning environments: 1) interest in the topic, 2) activities geared towards the topic, 3) students questioning and changing their thinking, and 4) the processes of transferring new information to other situations (Demircioğlu & Atasoy, 2006, pp. 19).

For example, *in the interest dimension,* worksheets were given attractive titles as well as cartoons, pictures, page decorations, puzzles and riddles. *In the activities stage,* worksheets were prepared on the topics included in the instructional process. The students were given instructions about these activities. They were also supplied with graphs and tables to transfer their new information. *In the questioning and changing of thinking dimension,* students could link new and existing information with the help of staged instructions. Finally, in the *transfer* processes, the new information was transferred into other situations. In order to do this, activities such as problem solving, poetry writing, collating a dictionary of geometry were used.

Studies in the Control Group:

In the Control group, the class teacher was informed about the purposes and stages of the study (pretest, posttest and retention test) prior to the pretest. She was guaranteed that she would not be told how to plan her instruction but it was mentioned that it would be essential for her to focus on the objectives specified during the class hours. As the study investigated the effectiveness of the instructional process, the focus in the control group was merely to follow instruction based on the objectives. Throughout the study, contact was maintained with the class teacher.

The class teacher in the control group taught her course by using the learning activities included in te guidebooks recommended by the Ministry of National Education. During and after classes, students completed activities given in exercise books. The teacher did not make effective use of mathematics teaching tools or materials.

This class teacher was observed in the instructional process for 10 weeks by the designer. The observations during these 10 weeks showed that the control group teacher remained loyal to the 24 objectives treated in the study as well as the 4th grade teachers' guidebook and students' text and exercise books approved by the Ministry of National Education.

Data Analysis

In the pretest, posttest and retention test stages of the study, "Geometry Level Identification Test" scores were interpreted by paired-sample t-test and "*One-Way Analysis of Covariance*" (One-way ANCOVA) (Büyüköztürk, 2010; Can, 2014). The analyses were performed on SPSS 15.00 package program. The independent variable in the study was instruction based on Dienes' principles, while the dependent variable was student success in geometry.

Paired-sample t-test was used in the study to identify whether there is a statistically significant difference between the mean data obtained from two consecutive measurements (pretest-posttest; posttest-retention) over the same data source (measurement tools) (Can, 2014, pp. 136).

When groups are not formed spontaneously and randomly in experimental studies, analysis of covariance (ANCOVA) needs to be used. The aim in using ANCOVA is to eliminate the covariates which have a relationship with dependent variables. Where there is a strong relationship between dependent variables and covariates and this relationship is removed from the analysis, ANCOVA yields more accurate results than ANOVA. This increases the correlations of measurements (Tabachnick & Fidell, 2012, pp. 19-20; 197). In an experimental study, if the researcher is examining whether a given procedure is effective or not, the most appropriate statistical operation to use is analysis of covariance (Büyüköztürk, 2010, pp. 112). In this study, Bonferroni correction was preferred as it is more sensitive than Sidak (Can, 2014).

Findings and Interpretations

Findings and Interpretations Regarding the Difference in the Pretest and Posttest Scores of Experimental and Control Groups

Prior to giving the findings regarding the first sub-problem, the ANOVA results of experimental and control students' geometry level identification pretest scores are given in Table 4.

Table 4. ANOVA Results of the Geometry Level Identification Test Pretest Scores of Experimental Groups 1 and 2 and the Control Group

Groups	N	sd	Mean of Squares	F	p*
Between Groups	353.123	2	176.561	.939	.395
Within Groups	15422.924	82	188.084		
Total	15776.047	84			

**p<.05*

The results of the one-way analysis of variance (ANOVA) conducted to identify whether there was a significant difference between the geometry level identification pretest mean scores of the study groups (Table 4) showed that no significant difference existed between the groups [$F_{[2-82]}$= .939, p> .05]. This means that the groups had identical geometry level identification test pretest scores.

The first subproblem of the study concerned whether there was a significant difference in the mean scores of "Geometry Level Identification" pre- and posttests within and between groups. In order to find this, analysis of covariance (ANCOVA) was used first to compare the mean scores within the groups. Data from the analyses are presented in Tables 5, 6 and 7.

Table 5. True Posttest Scores of Groups and Their Posttest Scores Corrected According to Pretest Scores

		Posttest*		Corrected Posttest	
Groups	***N***	$\bar{x}$	***s.error***	$\bar{x}$	***s.error***
Experimental 1	31	55.84	2.370	57.75	1.587
Experimental 2	29	61.07	2.293	59.43	1.637
Control	25	41.44	2.398	40.98	1.756

** The highest possible score from the geometry level identification test is 76.*

The "*Geometry Level Identification Test*" true posttest mean scores were 55.84 in Experimental Group 1, 61.07 in Experimental Group 2, and 41.44 in the Control Group (Table 5). The ANCOVA results showing whether the difference observed between the corrected mean scores of groups was significant or not can be seen in Table 6.

Table 6. ANCOVA Results on Geometry Level Identification Posttest Scores Corrected According to Pretest Scores

Source of Variance	Sum of Squares	sd	Mean of Squares	F	p*	η^2
Pretest (Regresyon)	8564.762	1	8564.762	111.151	.000	
Groups (Posttest)	5491.778	2	2745.889	35.635	.000	.468
Error	6241.454	81	77.055			
Total (Corrected)	262548.000	84				

**p<.05*

The ANCOVA results (Table 6) showed a significant difference between the geometry level identification posttest mean scores corrected according to the pretest scores, $F_{(2-81)} = 35.635$, $p<.05$. In other words, the method used in the groups affected student success. The impact size value (η^2) of the difference was .468. This means that the independent variable had a large impact on the dependent one (Cohen, 1977). In other words, instruction based on Dienes' principles had a large effect on geometry success in the Experimental groups.

The Bonferroni multiple comparison test was conducted to see which group was favored by the difference between the corrected posttest scores. The Bonferroni test results are shown in Table 7.

Table 7. Bonferroni Results of the Corrected Geometry Level Identification Posttest Scores

Groups	Groups	Difference between Mean Scores	s.error	p	Source of Difference
Experimental 1	Experimental 2	-1.682	2.239	1.000	Experimental1>Control Experimental2>Control
	Control	16.762*	2.370	.000*	
Experimental 2	Experimental 1	1.682	2.239	1.000	
	Control	18.444*	2.398	.000*	
Control	Experimental 1	-16.762*	2.370	.000*	
	Experimental 2	-18.444*	2.239	.000*	

**p<.05*

According to the results of the Bonferroni Multiple Comparison Test which was conducted to reveal the differences between the corrected geometry level identification posttest scores of the groups (Table 7), the success of the instructional process in Experimental Groups 1 and 2 ($\bar{x}_{Experimental\ 1} = 57.75$ and $\bar{x}_{Experimental2} = 59.75$), was significantly higher than that in the Control Group ($\bar{x}_{Control} = 40.98$). No significant difference was found between the success levels of Experimental Groups 1 and 2. The findings suggest that instruction based on Dienes' principles offered in the experimental groups by the class teacher and the researcher was more effective than the instructional process in the control group in reaching the objectives of the geometry and measurement (area and circumference) learning domains.

In another dimension of the first subproblem, paired samples t-test (paired samples t-test) was conducted to see whether a significant difference existed within the groups regarding "Geometry Level Identification Test" mean scores given as pretest and posttest. The results of these analyses are shown in Table 8.

Table 8. t-Test Results of Geometry Level Identification Pretest and Posttest Mean Scores of Groups

Groups	Measurement	N	$\bar{x}$	Ss	Sd	t	p*
Experimental 1	Pretest	31	28.55	14.22	30	-20.778	.000
	Posttest	31	55.84	13.92			
Experimental 2	Pretest	29	33.31	13.36	28	-21.630	.000
	Posttest	29	61.07	15.02			
Control	Pretest	25	31.72	13.47	24	-3.630	.001
	Posttest	25	41.44	10.56			

**p<.05*

Table 8 shows a significant difference between the geometry level identification test mean scores prior to (pretest) and after (posttest) the experiment in Experimental Group 1 [$t_{(30)} = -20.778$, $p< .05$]. In Experimental Group 2 too, there was a significant difference between the geometry level identification test mean scores prior to (pretest) and after (posttest) the experiment [$t_{(28)} = -21.630$, $p< .05$]. The control group also revealed a significant difference between the geometry level identification test mean scores prior to (pretest) and after (posttest) the experiment [$t_{(24)} = -3.630$, $p< .05$]. The difference in all three groups was in favor of posttest mean scores.

An increase was observed from the pretest to posttest mean scores of all three groups at the end of the instructional process. The increase was 27 points in Experimental Group 1, approximately 28 points in Experimental Group 2 and approximately 10 points in the Control Group. Therefore, the instructional process in the two experimental groups seemed more effective than that in the control group. In other words, instruction based on Dienes' principles may be said to be effective in student learning.

Findings and Interpretations Regarding the Difference in the Posttest and Retention Test Scores of Experimental and Control Groups

The second subproblem of the study concerned the potential presence of a significant difference between experimental and control students' mean scores obtained from the "Geometry Level Identification Test" used as a posttest and retention-test. To this end, analysis of covariance (ANCOVA) was performed first to compare the mean scores of the groups. Data from the analyses are presented in Tables 9 and 10.

Table 9. True Retention Test Scores and Retention Test Scores Corrected According to Posttest Scores

Groups	**Retention Test**			**Corrected Retention Test**	
	N	$\bar{x}$	***s.error***	$\bar{x}$	***s.error***
Experimental 1	31	48.77	2.345	46.34	1.443
Experimental 2	29	55.03	2.091	47.41	1.566
Control	25	34.80	2.531	46.66	1.779

The true retention test mean scores of the groups from the "Geometry Level Identification Test" were 48.77 in Experimental Group 1, 55.03 in Experimental Group 2, and 34.80 in the Control group (Table 9). ANCOVA analysis was performed to explore whether the difference observed between the corrected mean scores of the groups was significant. The analysis results are given in Table 10.

Table 10. ANCOVA Results of Geometry Level Identification Test Retention Scores Corrected According to the Posttest Scores of Groups

Source of Variance	**Sum of Squares**	**sd**	**Mean of Squares**	**F**	**p***	η^2
Posttest (Regression)	14578.669	1	14578.669	228.775	.000	
Groups (Retention)	16.866	2	8.433	.132	.876*	.003
Error	5161.715	81	63.725			
Total (Corrected)	211598.000	84				

*$p<.05$

According to ANCOVA results (Table 10), no significant difference was found between the retention-test mean scores of the participants corrected according to their geometry level identification posttest scores, $F_{(2-81)} = .132$, $p>.05$. The impact size value of the difference (η^2) was .003. The impact size coefficient reveals a small impact (Cohen, 1977). No difference in retention test mean scores in any group means there was no difference between students state or amount of forgetting learned information. In other words, there was no significant difference between the information retention levels of the three groups after three weeks as evidenced by their posttest mean scores.

In another stage of the second subproblem, paired samples t-test was performed to explore whether there was a significant difference between the mean scores of the "Geometry Level Identification Test" used at the end of the experimental process (posttest) and three weeks after the experiment (retention test). The results are shown in Table 11.

Table 11. t-Test Result of Geometry Level Identification Posttest and Retention Test Mean Scores of Groups

Groups	Measurement	N	$\overline{x}$	Ss	Sd	t	p*
Experimental 1	Posttest	31	55.84	13.92	30	7.280	.000
	Retention	31	48.77	15.92			
Experimental 2	Posttest	29	61.07	15.02	28	5.994	.000
	Retention	29	55.03	18.45			
Control	Posttest	25	41.44	10.55	24	2.764	.011
	Retention	25	34.80	10.40			

*$p<.05$

As shown in Table 11, there was a significant difference between the mean scores of the geometry level identification test used as posttest and retention test in Experimental Group 1 [$t_{(30)} = 7.280$, $p< .05$]. In Similarly, Experimental Group 2, a significant difference was found between the mean scores of the geometry level identification test used as posttest and retention test [$t_{(28)} = 5.994$, $p< .05$]. In the control group, where the effects of the activities in the Ministry of National Education’s guidebook on student success was investigated, a significant difference emerged between the mean scores of the geometry level identification test used at the end of the experiment (posttest) and three weeks after its completion (retention test) [$t_{(24)} = 2.764$, $p< .05$].

In all three groups, students forgot some of the learned information in the three weeks following the instructional process. In Experimental Group 1, there was a 7-point regression in the retention test over the 17 points gained in the posttest. In Experimental Group 2, six out of the 18 points gained in the posttest were lost. In the Control Group, approximately 7 out of the 9 points gained in the posttest were lost. Even though some information was lost to all groups, experimental students remembered most of the information they learned. In the control group, however, students regressed back to their pretest mean scores and forgot almost all the information they had learned in the three weeks following the study. In sum, all groups revealed a decrease between 5-7 points in retention mean scores as compared to posttest mean scores. This suggests that if the instructional process is not maintained, there may be problems in retention of learning.

Considering the findings from all subproblems of the study, it may be stated that learning activities based on Dienes' principles affect student success positively. It may be argued that the learning experiences offered to control students were not as effective as those offered to experimental students. Retention test findings, on the other hand, revealed that even though a significant difference did not exist between the mean scores of the three groups, experimental groups retained the information more than the control group.

Conclusion, Discussion and Recommendations

The present study investigated the effects of geometry activities based on Dienes' principles on student success and retention of learning. The findings showed that learning environments based on Dienes' principles were more effective than those based on guide, text and exercise books. In other words, the changes in the geometry success scores of students in both experimental groups were significant as compared to the change in the academic success scores of control students.

There may be two reasons why instructional environments based on Dienes' principles significantly affect students' academic success. The first reason is that Dienes' principles are thought to provide children with opportunities to learn mathematical concepts over many diverse experiences. In Dienes' learning environments, students experience processes such as starting the mathematical process with play, use of manipulative materials, active physical and mental involvement, maximum concept development experiences (the perceptual variability principle) and revealing the relevant/irrelevant aspects of concepts (the mathematical variability principle). Evidence that Dienes' principles facilitate the effective learning of mathematical concepts parallels other findings in the literature (Gningue, 2000, 2006; Sriraman & English, 2005; Velo, 2001; Zhang, 2012). For instance, in a study with secondary school students, Gningue (2000) concluded that instruction based on Dienes' variability principles helped enable students to succeed in algebraic operations. Another study by the same author used Dienes' perceptual and mathematical variability principles to teach the concept and processes of equation, and reached success in both groups (6th and 7th grades). Classroom learning reached a success level of over 80% (Gningue, 2006).

Similarly, a study by Zhang (2012) revealed that activities based on Dienes' principles changed children's fraction concept images positively. The learning environment in line with Dienes' principles led to an increase in students' test performance, enrichment of their fraction concept images, and development of their conceptual understanding. Studies by Sriraman and English (2005) and Velo (2001) also corroborate the findings at hand regarding the significant effects of Dienes' principles on student success.

The second reason for the increased success of both experimental group students may be the constructivity principle, which is the core of Dienes' principles. With the "constructivity" principle, the environment is prepared by the teacher as students construct their mathematical concepts and information. By the end of the learning process, students reach a principle, rule or formula by themselves. As in the existing study, it increases students' mathematics success, develops their higher order thinking skills, and positively affects their interest and attitudes in the mathematics course (Arseven, 2010; Ayaz & Şekerci, 2015; Çağlar, 2010; Çilingir, 2015; Işık & Çağdaşer, 2009).

On the other hand, control students may not have been as successful as their experimental counterparts as they did not experience Dienes' principles in the classroom environment. In other words, interviews with and observations of the control group teacher suggested that students in this group may have been less successful as their instruction did not involve play for the abstraction of mathematical ideas, manipulative materials were not used sufficiently, physical and mental involvement was not active, maximum experiences of concept development could not be achieved, relevant and irrelevant qualities of the concept could not be clarified, and the students were not given enough opportunities to construct their knowledge. Toptaş (2008) study showed that even in the first grade, teachers used very few concrete materials in geometry instruction, did not let students do the activities themselves, and did not allow them to learn by doing. In addition, Toptaş (2008) claimed that if the teacher consistently goes over the activities in the mathematics guidebook, textbook and exercise books in a controlled manner, this denies students the opportunity to reveal themselves. Therefore, it is worth noting that Toptaş emphasized that these activities did not serve their purpose; students did the learning activities only if they were asked to: and they turned geometric shapes study into a coloring activity not covered by the learning objectives (Toptaş, 2008).

As elementary geometry education affects students' future education, it may be expected that students in the control group will experience problems in geometry in the future. In other words, the high correlation between elementary school success and success in future years, especially starting from grade three (Bloom, 2012), may mean that failure in national and international exams may be blamed on elementary school. However, high quality instruction contributes to students' understanding of the geometry course and a robust construction of geometry concepts in their minds. For this reason, the instructional process in the study was based on constructivism, starting mathematics classes with free play followed by semi-structured and structured activities. The constructivity experiences emphasized by Dienes formed the backbone of the success of the experimental groups in the study because his principles offer maximum experiences in concept development and the construction of mathematical ideas. Particularly in geometry, presenting models in various sizes and positions enables students to construct geometric concepts (Toptaş, 2010).

Similarly, Dickson et al. (1984) emphasize inadequate education among the reasons for failure in geometry, and state that presenting children with a single shape in different positions makes them think that the shape has changed. They blame this on the fact that shapes are always presented in one way. Therefore, when children face non-standard situations, they have difficulty generalizing the concepts. Dienes' perceptual variability and mathematics variability principles may eliminate such problems by presenting children with shapes in different directions, positions and sizes, while at the same time protecting their unchangeable qualities.

Results regarding another subproblem of the study concern the retention test. Three weeks after the completion of the instructional process, both experimental and control groups took a retention test. The mean scores from these tests showed that all groups forgot some of the information at a significant level. In all groups, retention test mean scores fell 5-7 points as compared to the posttest mean scores. Even though a significant difference did not exist between the retention test scores of the groups, it may be stated that there was more retention of information in the experimental groups than in the control group. While in the experimental groups, the 18-19 points of increase in the posttest scores fell by 6-7 points, the control students forgot approximately 7 points out of the 9 points of increase in the posttest. Consequently, it may be claimed that control students forgot almost all of the information they learned.

While several experimental studies on elementary mathematics education have concluded that instructional methods affect children's retention scores (Altunay, 2004; Kurt, 2015; Yücel Yumuşak, 2014), others have concluded otherwise (Yiğit, 2007). Even though the experimental groups in this study followed games and manipulative materials and went through a more constructive process than the control group, the significant fall in retention test scores may be attributed to several factors:

To begin with, students in the experimental groups had learning experiences other than Dienes' principles in the four weeks following the completion of the experiment. In other words, the experiment in the instructional process was over. The instructional approaches that teachers used afterwards, moving on to another learning domain of mathematics after the experimental process and a possible lack of linking between learning domains may have led to the forgetting of some of the information. Secondly, factors such as the experiment taking place later in the school year and end-of-term fatigue may also have led to forgetting on the students' part.

Another reason may be the limited time frame for the instructional process in the experimental groups. Tertemiz (2005) states in her study on the teaching of numbers that the instructional process in experimental studies bring temporary learning and that the instructional process must be sustained for a longer time for the difference in favor of the experimental group to continue. As the literature contain s no evidence for the effects of Dienes' principles on information retention, it is believed that both the present study and any future ones on Dienes' principles will contribute significantly to the literature.

The present study has several limitations. Previous studies on Dienes' principles do not offer information about the permanence of information learned in instructional environments based on these principles. Therefore, retention test scores from the experimental groups in the present study could not be compared against findings from other studies. Another limitation of the study has been that students' geometry success was measured by multiple choice tests designed by the researchers. Therefore, the effects of learning environments based on Dienes' principles on students' higher order thinking skills could not be clarified.

Based on the results of the study, the following recommendations may be made:

- The present study only focused on 4th grade learning objectives. As most objectives were at information and concept levels and rarely at implementation level, children's higher order thinking skills, which are important in the constructivist approach, could not be adequately measured. Future studies may examine their assumption skills, preferred strategies, etc, by using non-routine problems at implementation level and above.
- The present study focused on geometry and relevant measurement areas. In order to raise the validity of the results, the effectiveness of Dienes' principles may be tested at different learning domains and grade levels.
- Another topic worth studying would be the reasons for the retention results obtained in the experimental groups. Even though the retention levels in both experimental groups may have been higher than the control group three weeks after the completion of the instructional process, there was a certain level of forgetting (5-7 points). Testing retention again with longer intervals in future studies about Dienes' principles may clarify this issue.
- Future studies may investigate the effects of using play and manipulative materials while implementing Dienes' principles on students' affect regarding mathematics (attitude, motivation, anxiety, etc.).

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