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Analysis of 8th Grade Students' Number Sense Related to the Exponents in Terms of Number Sense Components *

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Abstract

The aim of this study is to analyze 8th grade students' exponential number sense in terms of number sense components. The research was carried out by twenty 8th grade students of a in a city center of Aegean region. The exponentials pairs comparison test was used to determine the interviewees. Data of the study were collected with Number Sense Scale on Exponentials. The data obtained from interviews were analyzed by using qualitative techniques. The result of the research showed that the use of numbers sense of the 8th grade students were quite low. In addition, students tended to use standard procedural solutions of short, time consuming and practical methods. Structure of question has emerged as an important factor in determining the use of number sense. Students used the equivalent representation components more successfully than the other components. But they used the other components quite unsuccessfully. Especially the research showed that students were inadequate in understanding the effects of operations on numbers less than 1.

Keywords

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Introduction

Is $72 \div 0{,}025$ bigger or smaller than 72? Is there any number between $\frac{2}{7}$ and $\frac{3}{7}$? A student asserted $72 \div 0{,}025$ is smaller than 72 (Yang, 2005). Another one thought that, there is no number between $\frac{2}{7}$ and $\frac{3}{7}$ (Markovits & Sowder, 1994). What is the sense which makes these students doubt about their thinking? What skill may provide these students answer these questions without doing standard and mechanical procedure? This sense is labeled as the number sense by National Council of Teachers of Mathematics (NCTM) in Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). In addition to this, in Number Sense and Numeration Standard for grade K-4 The NCTM Curriculum and Evaluation Standards (1989) asserted that children with good number sense "(1) have well-understood number meaning, (2) have developed multiple relationships among numbers, (3) recognize the relative magnitude of numbers, (4) know the relative effect of operating on

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number, and (5) develop a referent for measures of common objects and situations in their environment" (p. 38).

Number Sense Components

Researchers who worked on number sense made different classifications of number sense components (Greeno, 1991; Markovits & Sowder, 1994; McIntosh et all., 1992; Reys vd., 1999; Sowder & Schappelle, 1994). Among them the following components were considered in this study; equivalent expression, number estimation, number value, operation effect and using benchmark. Equivalent expression involves abilities of calculating pratically by using the equivalence of given number or expression. Number estimation means to think of an appropriate estimation and estimate approximate value. Number value involves comparing numbers, determining the closest number, ordering numbers and finding a number between two numbers. Operation effect involves the comprehension of the meaning and effects of the operations like understanding the effect of multiplication with a number less than 1. Using benchmark is the ability to do comparison by using friendy numbers and benchmarks to help decision.

Research on Number Sense

Some of the researches on number sense investigate the relationship between number sense and grade level, gender and mathematics achievement. These studies revealed that usage of number sense is very low level (Harç, 2010; Işık & Kar, 2011; Kayhan-Altay, 2010; Menon, 2004; Mohamed & Johnny, 2010; Singh, 2009; Yang, 2005); the structure of the question effects the usage of number sense like, challenging (Kayhan-Altay, 2010) and context based questions (Sturdevant, 1991) rather than standard questions leads to use number sense; compared to natural numbers students have difficulties on rational numbers and decimals (Kayhan-Altay, 2010; Mohamed & Johnny, 2010; Singh, 2009); there is a positive relationship among language, spatial reasoning, memory and number sense (Jordan, Glutting & Ramineni; 2009); number sense and mathematics achievement (Harç, 2010; Jordan, Glutting & Ramineni, 2009; Kayhan-Altay, 2010; Mohamed & Johnny, 2010; Sturdevant, 1991); and number sense is a strong predictor of later mathematics (Mohamed & Johnny, 2010). Although Kayhan-Altay (2010) and Mohamed and Johnny (2010) stated that the ratio of usage of number sense is getting less in later grades and the inclination of using standard procedure increase, Singh (2009), Pike and Forrester (1996) and Işık and Kar (2011) found that as the grade level and age increase students number test score increase. As Sturdevant (1991) asserted that students were more successful on the components of understanding meaning of operation compared to other components, Harç (2010), Mohamed and Johnny (2010) and Singh (2009) found correct response percentage on understanding the effects of operation component is lower. Harç (2010) found that students use number sense most in the measuring benchmarks component.

Menon (2004) found that students have incompetency in estimation questions. Research showed that students has faith in standard procedure and rules and they generally choose these methods in solving problems (Harç, 2010; Kayhan-Altay, 2010; Singh, 2009; Yang, 2005), besides students who preferred standard methods do not recall the rules correctly and answer with some personal generalization like 'multiplication makes bigger and division makes smaller' (Harç, 2010). While Singh (2009) and Kayhan-Altay (2010) found that the male students got higher score, Menon (2004) revealed that female participants got higher score. But these differences were not statistically meaningful within each grade. Harç (2010), Kayhan-Altay, (2010), Menon (2004) and Sturdevant (1991) stated that there is no statistically significant gender difference within grades. Only statistically significant difference detected in Singh (2009)'s study in which there was a difference for first grade students in favor of male students.

Research investigating relationship between number sense and some mathematical abilities (estimation, representation, written calculation) revealed that students written calculation performances are better than their number sense score (Reys & Yang, 1998; Yang & Huang, 2004); higher score on written calculation doesn't not accompanied by meaningful learning (Yang & Huang, 2004), students have faith on written calculation but when they arre encouraged they can used number sense (Reys & Yang, 1998), while there is no relation between measurement estimation and number sense, there is a strong corelation between area estimation and three number sense components (Pike & Forrester, 1996), student who can easily translate between different representations have high number sense (Yang & Huang, 2004), students with high number sense have high number solving abilities on nonroutine problems (Işık & Kar, 2011), students have difficulties in interpreting their procedures and results, they generally tended to make rule-based explanations (Işık & Kar 2011), students have difficulties in making connections between fractions and decimals (Reys & Yang, 1998).

Research in United States of America, Australia, Sweden and Taiwan on comparing number sense of different cultures displayed that students had inadequacies in connecting decimals and fractions (Reys, Reys, McIntosh, Emanuelsson, Johansson & Yang, 1999), in Isreal and Korea students were better on natural numbers compared to fractions and decimals, (Markovits & Pang, 2007), their level of benchmark usage is low (Markovits & Pang, 2007; Reys et al., 1999), there is no gender difference in number sense usage of Chinesse and Finnish students (Aunio, Niemivirta, Hautamaki, van Luit, Shi & Zhang, 2006), culture caused the difference related with number sense (Aunio et al., 2006; Markovits & Pang, 2007; Reys et al., 1999), inadequacy in number sense in a common problem in most of the countries (Reys et al., 1999), the emphasize on standard computations and finding exact numbers effects negatively to the usage of number sense (Markovits & Pang, 2007; Reys vd., 1999). For example although when Korean students, who educated with a program emphasize traditional calculations, were given change they could use number sense, without any guidance they had inclination to keep using standard calculations (Markovits & Pang, 2007).

On the other hand the research on comprehension of number and number misconceptions revealed that students have problems in estimating when comparing exponential numbers (Sastre & Mullet, 1998), it is necessary to make sense of the "repeated multiplication" approach on exponentials whose base and power are real number (Pitta-Pantazi et al., 2007), 13-14 year old students think that increase in exponential number value is additive (Sastre & Mullet, 1998), students are successful on exponential numbers whose base and power are natural numbers but they are less successful and have difficulties on comparing exponentials whose base and power are in different number areas (Avcu, 2010), students in different education levels have difficulties and misconceptions in exponential numbers (Avcu, 2010; Cengiz, 2006; Şenay, 2002), the reason of these difficulties can be unabling to determine the value of the exponential numbers, not making sense of zero as a power, not differentiate the expressions of $(-a)^n$ and $-a^n$, not making sense of negative power, not differentiate the expressions of x^n and n^x, not comprehending that when the power is even the value of exponential number is always positive, difficulties in finding the value of exponential numbers, difficulties in addition and subtraction of exponentials, difficulties in multiplication and division of exponentials, difficulties in operations of exponentials with negative power (Duatepe-Paksu, 2008).

Research Problem and Significance of the Study

Numbers and arithmetic is an important area of school mathematics. The presentation of these subjects may be limited to the rules of arithmetic operations and formal written calculation skills. Because of this limitation most of the people may percept mathematics as consists of rules and formulas. Making sense of the numbers, and their relationships with each other must be known to solve problems involving numbers. Number sense is one of the topics studied over the last 20 years and many researchers emphasized its importance.

In literature there is considerable amount of study on number sense from different countries. The different definitions of number sense involve the list of its components (Greeno, 1991; Kayhan-Altay, 2010; McIntosh et al. 1992; Reys et al., 1999). For this reason, in order to determine students' number sense, it will be significant to examine their responses to number sense components. Since number sense has begun to be searched recently in Turkey, there is not much research on this area.

Exponential numbers are frequently seen in elementary and later grades. Generally students have difficulties in this subject. No studies have found on number sense on exponential number forms. To prevent inadequateness in exponentials, determining exponential sense is important. Because of that, investigation of exponential number sense with a qualitative research tool will be significant for literature. Therefore the aim of this study is to analyze 8th grade students' exponential number sense in terms of number sense components.The research problem can be stated as; how is the 8th grade students exponential number sense in terms of number sense components?

Method

The qualitative reaserch method was used in collecting and analysing data since the aim of this study is to analyze 8th grade students' exponential number sense in terms of number sense components.

Measurement Tools

For data collection two measuring tools namely the exponential pairs comparison test and number sense scale on exponentials were used.

The exponential pairs comparison test

The exponential pairs comparison test, developed by Pitta-Pantazi, Zachariades and Christou (2007), was used to determine the interviewees. This test has 20 items. Each item involves a pair of exponential numbers and ask to compare their values. Students were expected to use <, >, or between these exponents without using computers or calculator. The exponents in the items are not easy to calculate. The aim is instead of calculation, leading students to make comparison using the properties of exponential numbers. Besides students were asked to write what properties they used and their thinking for each item. The first 8 items involves the pairs who bases are the same but power are different, the remaining 12 items involves exponentials whose bases are different but power are the same. The bases and power of the exponentials are negative, positive and decimals in the test.

The last 4 items of the test involves the power in fraction forms. Since this type of exponents was not covered in the classroom, the only first 16 items were used.

Exponential Number Sense Scale

In order to determine the number sense components the students use Exponential Number Sense Scale was developed. This scale involves the components which are named in the literature as equivalent expression, number estimation, number value, operation effect and using benchmark. During test construction process the content validity has satisfied by the expert opinion scores. The scale has totally 11 questions in two parts (Appendix 1). The first two items in the scale is for equivalent expressions and using benchmarks. The third and fourth items are for only equivalent expressions components. Fifth and sixth items are on number estimation, item number seven and nine is on number value, item number ten and eleven are on operation effect. The eighth item is on both number value and using benchmark.

Data collection

The exponential comparison test developed by Pitta-Pantazi et al. (2007) was used to determine participants. This scale was administered 108 eighth grade students in a school in a city center of Aegean region in 2011-2012 academic year in second semester. The instrument was administered in common counseling hour (40 minutes) of all 8th grade sections. During administration the classroom teachers of each class was present in the classes. Besides that the first researcher were visited each class during administration to explain the aim of the study and answer the students' questions.

The total score of each student was determined by giving 1 for each correct and giving 0 for each incorrect answer. Then the difficulty index of each question was calculated. Items were categorized as the very easy, easy, medium, difficult, and very difficult. The interviewees are the students who got the score 8 and more. The aim of this selection was not to involve the students who were so weak on exponential numbers.

The study was implemented on 20 eight grade students on the first and third week of December 2011-2012 academic year. Interview dates were arranged with school administration by considering students schedules. Interviews were implemented in one of the room for special need students provided by school administration. All students were interviewed by the first researcher. Each student were shown the questions and told the aim of the interviews. They were explained that their names will not be published anywhere and their responses will not affect their grades. Interviews were audiotaped with the consent of the students. Students were assured that they can finish the interview anytime they want and if they want their record will be destroyed. Interviews were actualized in a friendly fashion so that students can express their thinking easily. Each interview lasted about 35-40 minutes.

During semi-structure interviews students were given the scale shown in Appendix 1 and asked to answer the questions. The questions were asked to the students in the same order. Students were asked to think aloud. To understand their thinking process clinical interview method was used (Ginsburg, 1997). After their responses they were questioned as "What did you ask?", "Why?" or "How did you decide?" Beside that students who were tended long procedures were asked "Can you decide without doing these procedures?" and "Is there any other solution way?"

Participants

The research was carried out by twenty 8th grade students in the same school in a city center of Aegean region. The exponentials pairs comparison test was used to determine the interviewees. Data of the study were collected with Number Sense Scale on Exponentials. The data obtained from interviews were analyzed by using qualitative techniques.

Data Analysis

Data gathered by interviews analyzed by qualitative techniques. Auidotaped records were transcribed in written forms. Students' written transcript and their works were filed separately. In each file students responses were categorized into subdivision according to each interview questions. The aim of categorizing data in terms of questions rather that categorizing it in terms of students is to compare the same student's responses to different questions.

To determine repeated code and themes in students' responses contents analysis was used (Yıldırım & Şimşek, 2008). In data analysis 2 researchers who were graduate students in mathematics education, had qualitative research course and experienced in qualitative data analyses were recruited. The responses of students who used number sense and who did not use number sense were analyzed in details. Data decoding process along with content analysis explained as follows. Data were examined by three researchers independently to identify the important expressions in terms of number sense. Then the researchers gathered together and grouped the identified expressions in subtitles. Each group was assigned a code. Codes were represented with a letter. Then data were examined and coded by each researcher. For coding reliability researchers gathered again and discussed their codes. Codes on which the researchers agreed on 100 % used in the themes. By assorting the codes of each question in common titles themes were created. Data organized according to themes were presented with supporting excerpt. In data analysis different codes and themes were created for each question. However when the repeated code were appeared for different question the same codes were used.

Work on Validity and Reliability

Participants, the interview atmosphere, data collection and data analyses techniques were explained in a detailed way. Besides the method of determining participants, data recording and organizing also explained in a detailed manner. In addition to that data were analyzed by the first researcher and two more researchers and only the codes with % 100 agreement was involved for analyses.

Results

The Analysis of Correct and Incorrect Responses and Usage of Number Sense

The distribution of correct and incorrect responses of each question is displayed in Table 1.

Table 1. The Distribution of frequency and percentage of correct and incorrect responses

Question	1	2a	2b	3	4	5	6	7	8	9	10	11
Correct	15 75%	13 65%	10 50%	15 75%	16 80%	1 5%	4 20%	10 50%	13 65%	14 70%	12 60%	10 50%
Incorrect or blank	5 25%	7 35%	10 50%	5 25%	4 20%	19 95%	16 80%	10 50%	7 35%	6 30%	8 40%	10 50%

The most correctly answered question was the fourth question which was related with equivalent representation component. The correct response rate was 50 % and more, except for questions 5 and 6. For the question 5 and 6 the estimation component may also be used. Only 1 and 4 students were answered the questions 5 and 6, respectively. It can be said that students displayed low performance in the questions of number estimation.

The number of students used number sense for each question was displayed in Table 2.

Table 2. The distribution of students who used number sense in solving interview questions

Question	1	2a	2b	3	4	5	6	7	8	9	10	11
Usage of number sense	3	3	1	15	18	5	14	10	11	13	12	10

As seen in Table 2 the number of students who used number sense is less for the question which can be solved easilyby standard procedures (question 1 and 2),. Similarly, the third question was on equivalent representation however it was not easy to use standard procedure for this question. This can be the explanation of the higher usage rate of number sense for this question.

How do you get the 52 values using three of the following?

$(\frac{1}{2})^{-4}$, 2^2, $(\frac{1}{6})^{-2}$, 52^0, 3^3, 1^0, 26, 2^{-1}, 5^2, 2

Figure 1. 4. Question

In the same way, the number of students who used number sense was higher in fourth question (Figure 1). Besides it is remarkable that the number of students who used number sense is higher than the students who responded correctly. The reason for that case is the responses which did not obey the giving criteria were accepted as incorrect. For example, a student made up 52 as 26x2 in a correct way however this calculation did not obey the criteria of using 3 numbers. However since the student got the number with its equivalent representation, it can be said that she used number sense.

$5 \times 10^6 + 10^{-6} - 10^6$
indicate the approximate value of this expression with only a number.

Figure 2. 5. Question

Similarly, the number of students who used number sense is higher than the students who responded correctly for the question 5 (Figure 2). Three of the 5 students who used number sense comprehended the number size. For example, for the value of 10^6 a student used the expression "very big number" and for the value of 10^{-6} she used the expression of "very small number". However she could not reached the correct answer. Another student made a guess, but his guess was not close enough to the correct answer.

$3^3 \times 2^2$ and $3^2 \times 2^2$ are two estimates for $3^3 \times 2^3$.
Which of these is closer to the $3^3 \times 2^2$? Why?

Figure 3. 6. Question

Lastly, for the question 6 (Figure 3) the number of students who used number sense is higher than the students who responded correctly. Only 14 of the students who used number sense could reach the correct answer. 12 students used number sense to decompose the number but because of incorrect interpretation their responses were incorrect. For example one of the students responses was as follows; *"Now here [$3^2 \times 2^2$] it reduced the value of three. For me it reduced too much. That is since it decreased the value of bigger one it decrease considerable amount. Here [$3^3 \times 2^3$] it is tiny that is since two is less than three, by making it bigger ones make smaller mistake."* It can be said that this student realized the between expressions of $3^3 \times 2^2$ and $3^2 \times 2^2$ the first one is 3 times the second one and it is bigger than the second one. Besides between the expressions of $3^3 \times 2^2$ and $3^3 \times 2^3$ the first one is smaller and the second one is 2 times of the first one. Up to this stage the student used number sense in order to compare the numbers by considering factors of them however because of not considering the idea of multiple they reached the incorrect answer.

Analysis of Usage of Number Sense in Terms of Number Sense Components

This study, which is related to the exponential numbers sense, yielded opportunity to see students' understanding of natural, rational numbers and integers. Students were good at transformation involving natural numbers. They most often used natural numbers and they felt most comfortable when making transformation with natural numbers. For example, in question 4 students more often preferred to exponential numbers represented by natural numbers in base and exponents. Another example, they were more successful in understanding of the effects of operations on natural numbers. However, very few students were able to comment about the effect of multiplication and division operations with a number less than 1. Also, the first four questions is actually related to equivalent representations components.

When we compare $1254 \div 12^{-21}$ and 1254×12^{-21}, which results of these operations is larger?

Figure 4. Question 10.

Equivalent representations component were used in other question by students (Figure 4). For example question 10 and 11 was prepared to examine understanding of the effects of operations. $1254 \div 12^{-21}$ was written as 1254×12^{21} and 1254×12^{-21} was also written as $\frac{1254}{12^{21}}$ by a student. Then; the student said that the first statement is greater than the other comparing expression of 1254×12^{21} and $\frac{1254}{12^{21}}$. As can be seen, students used the equivalent representation component because they felt more comfortable with exponential numbers represented by natural numbers in base and power. It was observed that students could not apply the rules for exponents correctly. For example, in the question 9, a student used an expression as "-3 times to make multiplication for 7^{-3}. Some students tried to make *predictions* in questions about approximate value. For example, a student's response is given in Figure 5.

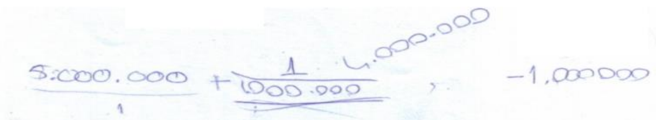

Figure 5. Student's response to question 5

The student wrote the expression 10^{-6} as $\frac{1}{1000000}$ and he said the results would be about 4 million. He made the following statement *"Now I've found as a result four million. Then here's* $\frac{1}{1000000}$ *.About 4 million. (after a while). I have to make the denominator equal to find the results".* The student thought prediction was not enough and he wanted to get common denominator. It can be said that predicting ability of the students was low.

$21^{-3} \square 31^{-2}$ place on the box one of the ">", "<" or "=" signs

Figure 6. Question 8.

In addition, we can say that this low ability is related to reference point. Students used the reference point in question 1, 8 and 9. In question 1 and 9 only 2 students and question 8, 8 students used reference point. Although eight students used reference point for the question 8 but more than half of the students were unable to do so. Compared to other questions, the reason of the increase in usage of number sense may be the structure of the question. Question 1, which relates to the usage reference point, it is appropriate to operation from the mind or to operation with paper and pencil. As well as question 8, 21^{-3} and 31^{-2} values are not calculated easily. It is reasonable to choose respectively 20^{-3} and 30^{-2} values instead 21^{-3} and 31^{-2}. For this reason, students *"I rounded* $\frac{1}{21}$ *to* $\frac{1}{20}$ *and this was equal to* $\frac{1}{400}$*. We will more multiply with* $\frac{1}{21}$ *value and and more will get smaller. I round also* $\frac{1}{31}$ *to* $\frac{1}{30}$*. I multiply 30 with 30 values, 900".*

$4^{-1}+0{,}76\,\square\,2^{0}$ Approximate value of the expression on the left and right of the box considering, Place on the box one of the signs ">", "<" or "="

Figure 7. Question 1.

Students have insufficient understanding related to *number magnitude*. For example, for students it is not surprising that the sum of $\frac{1}{4}$ and 0,76 is smaller than 1 (Figure 7). It might because students think the numbers as meaningless objects instead of thinking as a quarter or half. In other words; the students have no idea about the accuracy of the results of operation because of insufficient understanding related to *number magnitude.* In addition, Some students could not compare the numbers to each other despite understanding the magnitude of the number such as 10^{5} and 10^{-5}. For this reason, we can say that it is important to understand the magnitude of a number according to other numbers.

Discussion, Conclusion and Suggestions

Result of the research showed that use of exponential number sense of students were low. This result is consistent with findings of many studies were conducted in different countries (Harç, 2010; Işık & Kar, 2011; Kayhan-Altay, 2010; Menon, 2004; Mohamed & Johnny, 2010; Reys et al., 1999; Singh, 2009; Yang, 2005).

It was observed that basis of insufficient use of exponential number sense was found to be lack of students about integers and rational numbers. Indeed to prevention of misconceptions about the exponential numbers, Duatepe-Paksu (2008) has been stated that need to correct the defficiences related to integers and rational numbers.

Another finding of the research is that students prefer long operation instead of short and practical methods. Students chose standard operation and memorized rules where it is appropriate to structure question. The result are consistent with findings of several studies has been carried out (Harç, 2010; Işık & Kar, 2011; Kayhan-Altay, 2010; Reys & Yang, 1998; Singh, 2009; Yang & Huang, 2004; Yang, 2005).

The research has revealed that structure of the question to determining the use of number sense, is an important factor. Students are beginning to think about question that can not be solved by standard methods and they are directed to look for different methods. This result is consistent with other work (Sturdevant, 1991).

Another result of the research is that students have failed to predict and they do not trust the results obtained with guess. This result is consistent with findings of several studies indicating that students have insufficient information (Kayhan-Altay, 2010; Menon, 2004; Reys et al., 1999).

Students have been able not to comment about the effect of multiplication and division operations with a number less than 1, instead of this, they they have tended to application rules. This result is consistent findings of studies that related to understanding the effects of operations and carried out in Turkey and Malaysia. However students found to be more successful on the effects of operations at study that conducted by Sturdevant (1991) in the United States of America. these two results are inconsistent with each other. as it may be due to many reasons such as differences in curriculum, severity rating of standard calculations, the culture of the country in which the students found (Aunio vd., 2006; Markovits & Pang, 2007; Reys vd., 1999).

Another outcome of the study is that students have a better understanding of the effects of the operations because they feel more comfortable with exponential numbers represented by natural numbers in base and exponents. This result is consistent with findings of Avcu (2010) studies indicating that students are more successful about mental comparisons containing natural number.

Research has shown that students can not sense the number especially the exponential numbers with very large and very small value when they don't make the standard calculations. This result is consistent with study conducted by Sastre and Mullet (1998). Sastre and Mullet (1998) stated that Students in the 13-14 and 16-17 age group often fail to predict the value of the exponential. Students are very forced especially very large and very small numbers. Students tend to application mechanical operations. In other words, students have difficulty understanding growing values of numbers. Another similar case is that Pike and Forrester (1996) stated that children in 6-11 years about understanding magnitude of numbers 1 to 100 are more successful than 1 to 1000.

Students failed to comprehend the meaning of the increase of exponential. They think They think that it is an additive increase. This finding is consistent with study conducted by Sastre and Mullet (1998).

The students have made a successful transformation in expressions including exponential multiplication, however they have limitations in comparison of 3 different expression. This result is consistent with study conducted by Singh (2009). Singh (2009) stated that student failed to and can not comment about comparing equivalent expressions and they tried to do the calculation.

This study revealed that students have an inadequate understanding about rational numbers. These deficiencies has caused such as the implementation without understanding of the rules and having difficulty about comments and controlling of obtained results. There are also different research indicating that similar difficulties especially in rational and decimal numbers (Kayhan-Altay, 2010; Markovits & Pang, 2007; Mohamed & Johnny, 2010; Reys & Yang, 1998; Reys vd., 1999; Singh, 2009).

it was observed that the students make mistakes in exponential number with negative exponents such as $a^{-1} = -a$, $a^{0} = a$ or $a^{-1} = a^{1}$. It is pointed out that students have so mistake by previous researchers (Avcu, 2010; Cengiz, 2006; Duatepe-Paksu, 2008; Şenay, 2002).

Number sense components should be given more space in curriculum.

The sample of research were 8th grade students have confronted with operations with exponents the first time the year in which the study was conducted. The same study can be done with the 9th and 10th grade students by the researchers working on mathematics education. In addition development of number sense can be examined on the different forms of number with an experimental study.

Students should be given a chance to build and interpret mathematical knowledge with peer interaction and discussion. It should be included understanding the exponential number and operations in the lessons. Students have insufficient information about especially the effect of operations. Students should be encouraged to make comments about expressions including different forms of number without calculation.

The results obtained with the estimation is unreliable and worthless for students. Related with, it can be said that prediction abilities of the students should not be blinded and should be emphasized that this is an important skill.

Students made various mistakes about exponential and operations relating to the exponential. Sometimes students reached the correct answer with these mistakes. In order to realize these, measurement and evaluations should be done byincluding open-ended questions and the class discussions.

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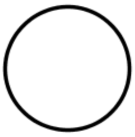
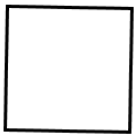
Appendix 1. Number Sense Scale Related with Exponential Expressions

1) $4^{-1} + 0{,}76 \, \square \, 2^{0}$ Place on the box one of the signs ">", "<" or "=",considering approximate value of the expression on the left and right of the box.

2) Shade the following figures, considering the results of operation

$4^{-1} + 2^{-1}$ $5^{0} - 3 \times 4^{-1}$

3) "$3^{6} \times 5^{9}$ has a bigger value than $3^{8} \times 5^{7}$." Is this statement true? How did you decide?

4) How do you get the 52 values using three of the following?

$(\frac{1}{2})^{-4}$, 2^{2}, $(\frac{1}{6})^{-2}$, 52^{0}, 3^{3}, 1^{0}, 26, 2^{-1}, 5^{2}, 2

5) $5 \times 10^{6} + 10^{-6} - 10^{6}$ indicate the approximate value of this expression with only a number.

6) $3^{2} \times 2^{2}$ and $3^{3} \times 2^{3}$ are the two estimates for $3^{3} \times 2^{2}$. Which of these is closer to the $3^{3} \times 2^{2}$? Why?

7) Which of the 2^{2} and 2^{10} values is closer to 2^{6}?

8) $21^{-3} \, \square \, 31^{-2}$ place on the box one of the ">", "<" or "=" signs.

9) 7^{-3}, 7^{-4}, 7^{2}, 7^{-2}, rank these numbers from small to large.

10) When we compare $1254 \div 12^{-21}$ and 1254×12^{-21}, which results of these operations is larger?

11) $175 \div 10^{-7}$ Which of the following is true for the result of this operation? Why?

- **a)** The result is much smaller than 175.
- **b)** The result is much bigger than 175.
- **c)** The result is slightly smaller than 175.
- **d)** The result is slightly bigger than 175.
- **e)** We cannot answer without operation.