Eğitim ve Bilim
2013, Cilt 38, Sayı 169

Education and Science
2013, Vol. 38, No 169

Following Students' Ideas: How Much to Let Go?

Öğrencilerin Fikirlerini Takip Etme: Nereye Kadar?

Recai AKKUŞ*
Abant İzzet Baysal Üniversitesi

Abstract

The purpose of this study was to find answers to the following two questions: What happens when a teacher follows his students' alternative ideas in his mathematics classroom? What is the limit of letting go in a problem solving process? A teacher with 15 years of mathematics teaching experience tried to modify his pedagogical practices towards an argument-based approach as part of a professional development project. This paper is a snapshot of a lesson selected from a number of videos recorded in his classroom when teaching "real numbers unit". The data were analyzed using an observation matrix whose bases are creating dialogic interaction, controlling problem solving process and making connections. The results revealed that the teacher hesitated to let the students follow their own problem solving process and explain their mathematical understanding because of his "comfort zone" in traditional way of teaching. This type of hesitation in changing pedagogy blocks shifting from an algorithmic view of mathematics to the mathematics as a constructed action.

Keywords: Teacher change, Student learning, Problem solving in mathematics

Öz

Bu makalenin amacı, şu iki soruya cevap aramaktır: Bir öğretmen problem çözme sürecinde öğrencilerin alternatif fikirlerini takip ederken neler olur? Problem çözme sürecini öğrencilere bırakmanın limiti nedir? On beş yıllık öğretmenlik tecrübesi olan bir öğretmen, profesyonel bir gelişim programı çerçevesinde pedagojisini argüman-tabanlı yaklaşıma göre değiştirmeye çalışmıştır. Bu makale, "reel sayılar ünitesinin" işlenişi sırasında kaydedilen videolardan seçilen bir dersten anlık bir fotoğrafı yansıtmaktadır. Veriler, temelleri *karşılıklı etkileşim yaratma*, *problem çözme sürecini kontrol etme* ve *bağlantı kurma* üzerine kurulu bir gözlem matrisi kullanılarak analiz edilmiştir. Sonuçlar, geleneksel öğretim yöntemindeki "güvenli bölgesi"nden kaynaklı olarak, öğrencilerin kendi problem çözme süreçlerini oluşturmalarına ve matematiksel anlamalarını açıklamalarına izin verme konusunda öğretmenin ikilemde kaldığını göstermiştir. Pedagoji değişiminde bu tür bir ikilem matematiği sadece algoritma olarak gören bakış açısından, matematiği insan ürünü olarak gören bir bakış açısına geçiş sürecini engellemektedir.

Anahtar Sözcükler: Öğretmen değişimi, öğrenme, matematikte problem çözme.

Introduction

The National Council of Teachers of Mathematics (NCTM, 2000) has focused attention on students' conceptual understanding of mathematics suggesting that students need to be actively involved in the learning process using their experiences and prior knowledge. Along with this view on learning, understanding of teaching has also been revised in mathematics classrooms. Teachers now need to provide students with a challenging and supportive classroom environment (Leikin & Kawass, 2005) in which they can build new knowledge by engaging in exploration of mathematical ideas by themselves. This change in the views of learning and teaching has placed students in the center of learning occurring in the classroom by altering students' roles and

*Recai Akkuş, Yrd.Doç.Dr., Abant İzzet Baysal Üniversitesi, Eğitim Fakültesi, İlköğretim Matematik Öğretmenliği ABD, E-mail: akkus_r@ibu.edu.tr

requiring them to be actively involved in talking and writing in mathematics classrooms. As learning is both a social and an individual act, students' argumentation of mathematical ideas with their peers and by themselves through their inner speech (Vygotsky, 1978) is crucial to supporting learning (Ernest, 1998).

Such a call for change in learning and teaching is not new in teacher education. NCTM (1991) identified four key areas for teachers' responsibilities: setting goals and creating tasks; managing classroom discourse; creating learning environment; and analyzing student learning for future instructional decisions. Also, Cuban (1993: p. 4) has already emphasized that fundamental changes needed to be applied to the classroom such that "teaching becomes structuring activities that enable students to learn subject matter, one another, and the community." Furthermore, Cuban has suggested that teachers will develop new ways of thinking about the nature of knowledge, teaching, and learning, and their actions in the classroom. Moreover, Sherin (2002) claimed that rather than using given practices, teachers must apply their existing understanding of teaching and learning when implementing reform education programs. Sherin further argued that teachers learn as they "negotiate among three areas of their content knowledge: their understanding of the domain, view of the curriculum materials, and knowledge of student learning" (p. 120). Along the same line, Lampert (2010:22) emphasized that "classroom teaching is relational work" as teacher is in relation to students, subject matter and circumstances (e.g., time, lesson plans, etc.) that bring them together.

The focus of this paper is on what happens when following students' alternative ideas and creating dialogical interaction among students during attempts to align pedagogical practices according to students' learning. For the changes that require students' active involvement in the construction of (mathematical) knowledge, teachers "have to take a step back in controlling students' learning activities" (Hoekstra, Brekelmans, Beijaard, & Korthagen, 2009:664). However, Wilson and Goldenberg (1998) have found that teachers hesitate to let their students explore mathematical ideas and solutions on a regular basis. In other words, teachers struggle to decide how much control of learning they should give to their students. Similarly, Brendefur and Frykholm (2000) have found that teachers, especially beginning or student teachers, struggle to create a dialogic interaction where students construct their own mathematical understandings. While students' communication of their ideas is an important concept in mathematics classroom, the attention should also be given to the construction of mathematical knowledge through *reflective communication* and collective argumentation (Brendefur & Frykholm, 2000; Cobb, Boufi, McClain, & Whitenack, 1997).

Analyzing the roles of the teacher in collective argumentation, Yackel (2002) suggests that argumentation is crucial to students' learning of mathematical concepts both as a collective and an individual act. The teacher plays an important role in initiating such an argument, supporting students' arguments as they interact, and supplying supports (data, warrant, and backing) that are omitted or left implicit in arguments (Yackel, 2002). Questioning is the key for setting up such an environment. Moreover, Walshaw and Anthony (2008), analyzing different research papers on the teachers' role in students' learning outcomes, have found that many research articles pointed out the importance of teacher's questioning in setting up appropriate classroom discourse for students' understanding of mathematics. In a research on instructional methods and strategies in science instruction, Treagust (2007) reported that the amount of classroom discourse is directly affected by teacher questioning and that higher level questioning has been shown to improve the amount and the quality of talk that occurs in the science classroom. Similarly, Martin and Hand (2009) highlight the importance of divergent questioning patterns of teachers in creating increased student voice, which is defined as the opportunity for students to engage in dialogical interactions with teacher and as well as in social context with peers.

Planning is the key for setting up such environments where students' voices can be heard and their ideas are valued. Leikin and Kawass (2005) and Simon (1997) argued that planning helps teachers set goals for learning in the classroom and evaluate their own understanding of the subject they are teaching. Simon (1995; 1997) disputes that this planning will be hypothetical because real learning occurrence will be different based on students' understanding of the subject and their earlier conceptions. Therefore, teachers need to understand the mathematics students bring into the classroom and to have the flexibility to support learning of the potential mathematics come out of classroom discussion (Ball, Thames, & Phelps, 2008; Johnson & Larsen, 2012; Shulman, 1986). As mentioned in the prior paragraph and as Leiken and Dinur (2007) and Simon (1995) pointed, teacher flexibility is important to make connections among the conjectures and ideas of students in a collective argumentation. Therefore, the role of teachers in a class discussion is to not only identify the potential mathematics but also create an environment based on students' ideas where students can construct their understanding based on the discussions. Leikin and Kawass (2005) attributed to Lampert (2001) that teachers should anticipate unexpected ideas from students and make overarching connections between students' thoughts and the subject they are teaching.

In the classroom, mathematical knowledge passes through a series of iterations of transformation as a result of student participation, as it does in the community of professional mathematicians—the iterative process of proofs and refutations (Borasi, 1992). In other words, the certification of students' personal knowledge of mathematics is analogous to the justification of objective knowledge in the domain of research mathematics. Yet, the teacher, as the authority over knowledge (expert authority) in the classroom (Amit & Fried, 2005; Weber, 1947), mostly determines whether students' construction of mathematical knowledge is "acceptable." Brodie (2011) attributed to Edwards and Mercer (1987) that students in schools learn to follow teachers' cues rather than to reason their answers to the questions. Furthermore, Pimm (1987) argued that teachers, intentionally or otherwise, use particular language to cue their students of "what to attend to" (p. 87). Learning of mathematics in schools is partly based on mathematical conversations structured by the teacher based on his/her own mathematical knowledge and on institutionally determined texts.

Furthermore, Simon (1997) has argued that the teacher, by avoiding being the authority of knowledge in the classroom, can promote students' negotiation of mathematical ideas and problem solution methods. This concept of dialogical interaction and negotiation of mathematical ideas and solutions gives students ownership of problem solving. Yet, changing their teaching style (giving up control of learning) is always a challenge for teachers. Emphasizing this challenge, Obara and Sloan (2009) have found that the change teachers saw in themselves and their work was different than the change observed.

This study was guided by the following two questions. What happens when a teacher follows his students' alternative ideas in his mathematics classroom? What is the limit of letting go in a problem solving process? Therefore, the purpose of this paper is to shed light on a classroom environment in which a mathematics teacher incorporated an argument-based approach in his classroom.

Method

Participants and Tasks

The data of this paper were selected from a year-long professional development (PD) project whose aim was to help teachers modify their pedagogies towards an argument-based approach in mathematics classrooms. During the PD programs the teachers in a public school in a rural area in the USA participated two five-day workshops (one in August, the second in February) that were organized in three cyclic phases. The teachers first experienced lessons structured around an argument-based approach as students, then had pedagogical discussions on the activities, and then prepared lesson plans based on argumentation. The teachers were required to teach one unit in the first semester and two units in the second semester using their plans they had prepared at the workshops. The teachers were observed during their implementation and feedback was provided by the observer. The selected teacher was a ninth-grade mathematics teacher with 15 years of mathematics teaching experience in different school districts in the USA. The class consisted of 18 students, one African-American, 17 White-Americans.

As for the data for this paper, George (pseudonymous name) taught a real numbers unit, chosen from the school textbook. The chapter consisted of addition and subtraction rules for real numbers, multiplication and division, and mixed review problems on the use of real numbers. George's students were engaged in small and whole group discussions along with writing tasks within and outside of the classroom. Some of the writing activities were writing an explanation of the real numbers (in classroom), writing a letter to parents about the real numbers, and writing a letter to a client of a construction company about the area of a rectangular ranch style house. The students also had the opportunity to read aloud in the classroom what they had written.

Data Collection

George's teaching was videotaped three times during this particular unit implementation. However, one of the three videotapes was chosen for the detailed analyses for this paper because

a) it was about midway in the project implementation;
b) it included a lesson with a writing activity in the classroom followed by a discussion;
c) it was the best representative of George's struggles while changing his teaching; and
d) it included sessions in which George followed his students' thoughts.

*Description of the lesson.*The lesson examined in this paper was about finding the cost of carpeting an L-shaped room similar to that shown in Figure 1. George told the students and wrote on the board that the price of the carpet was $30 per square yard. The teacher gave the students six minutes to work on the problem individually. As he was walking in the classroom, he answered the students' clarification questions about the problem. After six minutes, the teacher then asked the students their opinions about how they would solve the problem. Having discussed the possible solutions of the problem for about 20 minutes, the teacher decided to do a writing activity in the classroom. The students were then given five minutes to write about the real numbers that were used in the problem and also asked to read aloud what they had written in the classroom.

Prior to the lesson analyzed in this paper, George and I talked about the importance of questioning and he decided to work on his questioning in order to have the students more involved in the lesson. The observation of this lesson, like other observations, ended up with a debriefing session at which the teacher and I talked about the implementation. The focus was mostly on the teacher's pedagogical practices (e.g., how he managed students' alternative ideas). I specificially asked what was difficult about handling such ideas. After the debriefing, I filled out a detailed analysis of the lesson for my own records.

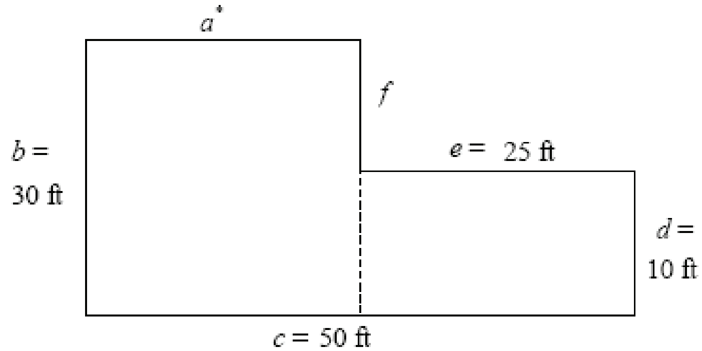

Figure 1: L-shaped room for carpeting (* the letters for the sides have been added for the analysis).

Data Analysis

The videotape of this lesson was transcribed. There were two main focal points for videotape analysis; the quality of classroom discourse and George's struggles during teaching especially when he follows students' ideas. "Classroom discourse" in this paper is used to refer to the communication or dialogical interaction between teacher and students and among students in terms of mathematics related conversations. An observation matrix adapted from Gunel (2006) was used to scrutinize George's pedagogical behaviors. There were three major areas in pedagogical practice that the matrix captures: creating dialogical interaction, controlling knowledge and the problem solving process, and unit preparation and making connections. The episodes (related segments of the lesson) were identified for each pedagogical area. Since the author of the paper was the only coder, after a month the segments were recoded from all over (Esterberg, 2002; McMillan & Schumacher, 1997). There was a 97% consistency between the before- and after-codings. Additionally, the types of questions that George asked were tallied and categorized into four: yes/no or factual type questions; higher-order questions that encourage students to provide further explanation for their ideas; computation questions that ask students to do the calculations; and no-answer questions which were asked by the teacher without expecting an answer or with teacher's self-answering [e.g., Fractions, decimals, would that just be? Part of the real numbers, right? Ok, any number on the number line.].

2.3.1. Implementation Criteria Matrix

The criteria matrix developed by Gunel (2006) was used to analyze the teacher's pedagogical behaviors in the classroom. The criteria matrix consists of three major areas in pedagogical practice. These criteria place a teacher in one of the four levels (*exploring, developing, transitioning, and practicing*) for defining the quality of classroom teaching. The levels are explained after the criteria.

Dialogical interaction is the first of the three criteria. Types of questions asked by teacher and students, teacher's response to students' answers and questions, and the direction of communication (e.g., from teacher to student) are considered as critical elements of dialogical interactions. The following conversation is an example of dialogical interaction between the teacher and a student:

T: *Twenty five feet. How did you get that, Nathan?*
Nathan: *'Cause fifty…minus twenty five is twenty five. …Ummm..*
T: *Ok, this big one down here is fifty, right? [Nathan: yeah] and this one is twenty five and we know this one plus this top one gives you the same as the bottom….*

Within the discourse analysis, I particularly focused on the types of questions the teacher used to create dialogical interaction among students. As mentioned above, types of questions were tallied and categorized. Creating dialogical interaction refers to the teacher's attempt to encourage students to have mathematical conversation with each other. The questions teachers ask in classrooms can promote or limit classroom conversation (*Why did you think we need the perimeter to find the cost of the carpet?*). The dynamic of dialogical interaction varies across the levels of implementation.

The second criterion, "*controlling*" criterion reflects an important step away from exploring level. This criterion is "*allowing students to take the responsibility for the thinking and problem solving process and moderating the conversation.*" Teacher domination of classroom discussion affects not only students' sharing ideas and reflection on other students' ideas but also the participation of students in classroom discussion. In the script above, George took the responsibility of solving the problem rather than letting Nathan continue to explain his thoughts. Finally, this criterion emphasizes allowing students to discover their own problem solving methods either as groups or individually rather than to provide an explanation of the teacher's own method.

Unit preparation and connection is the third criterion. Unit preparation refers to identifying the big ideas of the units which reflects teachers' understanding of the content knowledge. In deciding the big ideas, teachers are engaged in an inquiry about their students' prior knowledge on which students build new concepts. Making those connections requires centering the concepts of the units on the big ideas and students' prior knowledge and supporting students in learning mathematical language.

Teacher implementation level shows variation from exploring to practicing. A teacher in the "exploring" level represents more traditional teaching, whereas, a teacher in the "developing" or "transitioning" level shows improvement in questioning and creating dialogical interaction, and those teachers give students more opportunities to share their ideas and solutions for a mathematics problem. Teachers who have moved to the "practicing" level are better able to support dialogical interaction, help students connect everyday ideas to the mathematical big ideas, and react beneficially to students' unexpected ideas.

Results

In the following sections, I will outline the characteristics of the implementation according to the criteria matrix.

Creating Dialogical Interaction

After the students worked on the problem individually for six minutes, George started the conversation by asking the entire class "*What is the first thing that is most important for us to be able to solve this problem? What do we need to do?*" John, who is a major figure for the rest of the analysis, suggested *adding up all the sides*. The teacher followed John's idea, adding up all the sides after *finding* all the sides.

George, by asking the initial question above, prompted students to make a *claim* about the solution of the problem. The teacher took this opportunity (John's idea) to ask other students' ideas for finding the sides. He asked the entire class about the missing side (side *a*) at the top of the figure (Figure 1). There were five or six students volunteering to answer the question at the same time. The teacher called on a student, Nathan, at the front desk.

11 T: Twenty five feet. How did you get that, Nathan?

12 Nathan: 'Cause fifty …minus twenty five is twenty five.

13 T: Ok, this big one down here [showing the side c] is fifty, right? [Nathan: yeah] and this one [showing the side e] is 25, and we know that this one [side e] plus this top one [side a] gives you the same as the bottom [side c]. And this one here [side b] …[a student is raising her hand] Courtney?

George asked Nathan how he got the answer; however, he did not let Nathan fully explain his thoughts. The conversation with Nathan took only one turn since George started to explain what he perceived as Nathan's solution.

Later on, George tried to include students' ideas by both acknowledging what John said, and by also asking other students to confirm what John said: "*is that what John said*?" After the teacher's clarification of finding the perimeter (utterance 18), he asked John why he thought that they needed the perimeter, which started a conversation between the teacher and John. This is the phase where the teacher is asking the student his reasons. This justification question helped George to realize that John misunderstood the problem and so he suggested John reread the problem, which assisted George to identify the trouble.

18 T: Twenty, all right,...so, is that what John said? He said we need to find the perimeter. And to find the perimeter, we need to know all the sides. How many people think knowing all the sides is the most important thing that we need to do on this problem? [All students are raising their hands including John]. How many people think we

need to do the perimeter [emphasized by the teacher] in order to find how much the carpet will cost? [Only John raises hand]. Ok, John, you are on your own here on this one. Why did you think we need the perimeter to find the cost of the carpet?

19 John: Because it is 30 dollars per yard, and when you add the perimeter [inaudible].

20 T: Ok! You said 30 dollars per yard [emphasized by the teacher]. Ok, is reading for information important when we read this problem? Ok? Reread the problem and tell me how much the cost is for this carpet.

21 John [Rereading the problem]: Thirty dollars per yard squared.

22 T: Yard squared? [John: Yeah]. Is there a difference between yard and yard squared? [John: Yeah]. What does yard refer to?

23 John: Just like a straight line.

24 T: Okay, linear measurement, right? Or like the perimeter you said? Since you read that first and you thought it said 30 dollars per yard, you thought the perimeter was the key thing to do, right? [John: Yeah]. What do you think now?

25 John: I still think that …

Even though the teacher mostly asked rhetorical, short answer, or yes-no questions, there were instances where he pursued students' justifications, which led him to realize that John mixed "yard" and "yards squared" in the problem (utterances 20-22). The dialogical interaction was mainly between the teacher and a particular student in the form of questioning-response-questioning (or evaluation). Yet, he sometimes posed questions to the entire class (utterance 30) in order to get them involved in the conversation. Nevertheless, the talk continued between George and that particular student (Lori) in the same form of interaction with John. Here, George attempted to create a situation where students could negotiate their ideas; however, due to the nature of questioning and interaction George could not use this opportunity.

30 T: You added all the sides together and came up with one-sixty? [John: yeah]. Does everybody agree with him? [Several students mumbling: yeah]. If you haven't added yet, add it and see what you get...Ok, 160 feet, right? [Several: yeah]. One hundred and sixty feet.[walking to his desk]. 160 feet for the perimeter and it's still telling us that it's 30 dollars per square yard. … What are we gonna do? [A girl raises her hand]. Yes?

31 Girl [Lori]: We need to find the area.

32 T: We need to find the area? [Lori: yes]. Why do we need to find the area?

33 Lori: Because, becau…just because!

34 T: What word tells you that? [Walking to the board].

35 Lori: Yards squared.

After emphasizing that the square part in the problem "*tells us that we need to find the area*" George told students that knowing how to find the perimeter helped them to find the sides. As an instructional decision, this was an indication of why George let the students work on the perimeter. This observation was verified by George during the debriefing session. He asked students how they would *now* solve the problem. Lori suggested splitting the L-shaped room into two rectangles by extending the side *f* to the side *c*. The teacher pursued Lori's reasoning behind that idea. However, the conversation was only between the teacher and an individual student, which made the interaction limited to questioning-response-evaluation format. Not only did George take more turns, but he also had more time to talk rather than pushing students to express their ideas.

48 T: Feet square. OK? 'Cause we are dealing with area, these're gonna be squared terms, right? So, what's this part over here [showing the right part of the figure], this other region? What is our area? [waiting 5 seconds] We have a rectangle here, right?

49 St1: Two hundred and fifty.

50 T: Ok, ten times twenty-five is two-fifty, correct. And that's feet what? [St1: Square]. Is that also feet square? 'Cause we're dealing with area, is it gonna be square or just feet?

[St1: Feet square]. Feet squared, ok. So, what is the grand total of both of those [showing $750ft^2$ and $250ft^2$ on the board]?

51 Sts [several]: One thousand feet squared.

52 T: One thousand …feet squared….Ok, so, we know the area, and we have an answer in squared terms. But, …but we want what? What? [St2: Yard]. We want it in yards squared. So, how are we gonna go from feet squared to yards squared?

In the sequence of this transcript (utterances 48-52) the teacher basically followed different problem solving episodes (*what can I claim about the solution?*, *what did I do?*, and *what are my reasons?*). However, these episodes are prompted by George instead of by the students themselves. The types of questions mostly focused on the factual knowledge or procedural skills in calculations (43,9% factual type and 10,6% computational). Furthermore, such questioning led most of the students to not to be directly involved in the problem solving process. Analysis of questioning revealed that only 7,6% of questions (5 out of 66) George asked required the students to give richer and longer answers with their reasoning. Moreover, George used the questions which did not require an answer with the frequency of 25 out of 66 (37,8%).

Based on the implementation matrix used for identifying the quality of classroom teaching, George was placed in the "developing" level for several reasons. First, he primarily interacted with one particular student, and he typically addressed one student at a time. He rarely asked others what they thought about their peers' ideas, and, more importantly, when students tried to provide their reasoning, George often interrupted them. While it appeared the purpose of most of the teacher's questioning was to extract the factual information from the students, he did exhibit some willingness to consider students' ideas. For example, the teacher did pursue John's justification for perimeter and hold it as a possible source of important information. He also asked students, instead of telling, what was different about perimeter and yards squared. This was an indication of his willingness to improve his pedagogical skills.

Controlling Knowledge and the Process of Problem Solving

The students were given opportunities to express their thoughts; however, they were interrupted by the teacher during their explanations. As seen in the transcript (utterances 11-13 and 48-52), George, rather than mediating the conversation, controlled the conversation and took the responsibility for the thinking process. He articulated his own thoughts instead of asking, for example, Nathan to explain his reasons (utterances 11-13). He interrupted another student when she attempted to explain her ideas along with her justification.

Moreover, the process of problem solving was also controlled by George. He often told the students what they would do next, which I interpreted as controlling the problem solving process (utterances 48-52). Lori had suggested splitting the L-shaped room into two rectangles by extending the side *f* to the side *c*; George asked how this information – *splitting into two rectangles* – would help them. Lori, via the teacher's questions, said that they could find the area of the rectangles using "base times height" formula. The teacher continued with Lori's ideas. The focus of George's questions was on the mechanical part of the problem solving, calculations: "*…here is our base and here is our height, multiply these two together, what do we get?*"; "*Seven hundred and fifty what?* [Sts: Feet] *Feet?* [Sts: Feet squared?] *Feet squared. Ok, 'cause we are dealing with area, these are gonna be squared term, right?*" (utterance 48).

Having found the area of the L-shaped room (utterance 52), the teacher led the students through the process of how they would "go from feet squared to yards squared" by asking specific factual questions. A student uncertainly answered, "*Times three?*" The teacher then offered two options for this step: "*Multiplication or division?*" The same student first said "multiplication" yet, after looking at the teacher's facial expression, he changed his mind and said "division." George then asked a *directed question* in a way to indicate that division was the right choice, "*Ok, why divided by three?*" He accepted this step and asked for a justification. This could have been a good question to expose student thinking, but even when faced with the fact that at least three students still thought they were dealing with linear measurement, George did not stop to discuss this important issue. He simply asked questions to hear the word he wanted: "square." Then, he led the students through the "right step" in the problem solving.

In the end, George *continued* to control the problem solving process by answering his own questions: "*Are we done? …Go back and reread the problem, right?*" (utterance 62). This kind of teacher behavior directed the students to follow what the teacher says and to carry out the computational part of problem solving. Teaching in such a manner, in turn, resulted in the loss of several opportunities to address possible misconceptions that students might have, which is an area I would like to analyze next.

60 T: Right, we're dealing with square, so it's gonna be three squared. Very good. So we gotta divide by? [pointing at the girl] [girl: nine]. Nine, ok, ...so, divide all this by nine. That's gonna give us square yards.
61 Sts [Several]: Hundred and eleven, one repeating
62 T: Ok, hundred and eleven, point one repeating. Are we done? …Go back and reread the problem, right? Reading information, here.
63 John [interrupting]: Multiply by thirty.
64 T: Multiplying by thirty because we're trying to find, here, what?

George was between "*exploring*" and "*developing*" levels for controlling knowledge and problem solving process. There were times when he attempted to change his role in the classroom along with his questioning. For example, he started to accept unexpected ideas and pursue them (utterance 18). However, he often interrupted the students when they were sharing, which led them to wait for the teacher to tell them what to do. In this matter, George had the total control of the process of problem solving. There was no group sharing, but only individual students sharing their ideas with him, the teacher.

Unit Preparation and Making Connections

After the lesson, I debriefed with the teacher. George said that the big idea for this particular lesson was "having students to understand the difference between whole numbers and counting numbers." His reasoning for using the area example was that "since we deal with measurement of a length, we cannot use negative numbers or zero. I just wanted to emphasize that."

At the beginning of the lesson, after asking the students how they would solve the problem, George realized that John had a different view about solving the problem (John had offered *adding up all the sides* for finding the cost of the carpet). The teacher pursued John's justification, which led the teacher to realize that "reading-problem confusion" might exist (utterance 20). George decided that they would eventually need to find the sides. Therefore, George took this opportunity to reactivate students' prior knowledge about finding the perimeter and area of a rectangle and their alternative conceptions about unit of measurement. Even though George had John realize that they were dealing with area (utterances 22-23), John still thought that they needed to find the perimeter (utterance 25). Later George said "*Well then,…we'll go ahead and go with that.*" This means that the teacher decided to follow the student's "wrong way" of solution. Due to the nature of interaction (e.g., teacher-student), his questioning (i.e., rhetorical, short-answer questions), and his control of problem solving process, George could not use this chance to address those alternative conceptions.

Having solved the problem, George asked the students to write a letter to parents about the real numbers used in the problem with the purpose of having the students to think back on the problem and connect it to the real numbers. While a student was reading her letter, she emphasized that "*Ohh, we use …real numbers to solve the problem. And we* <u>*just*</u> *[*emphasized by the student*] use whole numbers, like, … to figure out the area.*" Another student, Zach, argued that there were also decimals in the problem, which was the area of the rectangle. After back-and-forth discussion on decimals, George said, "*Ok, so, we, in a way, kinda used some of the whole numbers, didn't we? We used all of the whole numbers except for zero, so therefore, what numbers did we really use? … On this problem, in a way we used the whole numbers but we didn't use zero*."Here George mislead students, possibly, to think that only whole numbers must be the dimensions of a rectangle.

To summarize, George was sometimes aware of such (mis)conceptions and tried to address them; however, due to his approach in questioning and his control of the problem solving process, students did not really grasp the idea behind the discussions that took place in their classroom. In addition, his intention of taking the responsibility for the thinking process during the problem solving prevented him from realizing and addressing students' misconceptions or misunderstandings. Therefore, I placed the teacher at the "developing" level for this criterion, *unit preparation and connections*. Yet, the teacher could also be placed down at the "exploring" level because of his dominance in discussions. Besides all above, during the debriefing, George raised his concern about following students' ideas: "You may follow them but you may not get any where, may end up with a dead-end." This concern was the result focused rather than the process focused.

Discussion

This paper was shaped around the three crucial criteria (dialogical interaction, controlling of knowledge and problem solving process, and unit preparation and connection). As Simon (1995) indicates, understanding learning as a process of individual and social construction helps teachers build a conceptual framework with which to align their teaching according to their students' learning. This form of teaching harmonizes the three criteria as the teacher and students engage in mathematics classroom. Teacher's understanding of mathematics and mathematics teaching helps teacher direct the dialogical interaction according to conceptual idea(s) identified earlier. However, creating dialogical interaction that leads students to argumentative discourse is possible only if teacher views mathematics as a social construction and individual negotiation rather than algorithms constructed independent of learner in advanced (Ernest, 1998; Simon, 1997; Yackel, 2002). One of the main stream areas that George struggled to change was shifting from algorithmic information transfer view of learning to the one in which mathematics is a constructed problem solving view of learning through classrom-negotiation and self-negotiation. Less number of higher-level questions might be because of this struggle.

In addition, teacher domination of classroom discussions does affect students' creation of ideas and participation in discussion. Students (and teacher) in traditional classrooms see teacher as the source of authority for mathematics (expert authority) (Weber, 1947); therefore, the teacher's control of knowledge and problem solving is accepted as normal (Amit & Fried, 2005). Even though George attempted to change his pedagogical practice, he was not able to give up the control of the problem solving process (i.e., *Are we done? Go back and reread the problem, right?*) (utterance 62). In such a scenario, the teacher seemed to be the only authority of knowledge deciding the amount of information needed and when to be delivered. As a result, the students relied on the teacher's existence during problem solving and did not have the responsibility for solving the problem except for doing the calculations. Consequently, the more teachers stick up to this kind of teaching views, the more they control problem solving process, and therefore the less dialogical interaction that leads students to argumentative discourse occurs in mathematics classroom.

What is it that George was comfortable in his "traditional" teaching? That is the comfort zone in which he did not need to deal with students' misconceptions and alternative ideas that confused other students. In his old tradition and in the control classrooms, he would have said the correct answer rather than digging into students' conceptualization. However, George started to panic as getting the right answer took more time. Therefore, he fell back in his old way of teaching. In other words, George, in pursuing specific information from students, comforted himself that the student "understood" or "knew" the topic. After receiving the correct answer (yards squared) (utterance 35), he confirmed it by repeating the answer in a full sentence (i.e., ok, the yards squared part tells you that we need to find the area).

George struggled to be more flexible in classroom discussion. There were many opportunities where students could have engaged in rich mathematical discussions. This inflexibility was about his obligation to the curriculum (Leikin & Dinur, 2007) rather than his lesson plan. Therefore, some factors influenced George's decisions in the moment. These factors (e.g., curriculum, students' confusion because of the vauge discussions) caused George to step back in implementing student-oriented lesson. His focus of the lesson was getting the right answer. Because of his dominance in classroom discussion and problem solving, the results of following students' ideas, according to George, ended up with a dead-end.

Conclusions

The analyses in this paper suggested that the teacher, due to his didactic approach, was keen to be the center of classroom conversation rather than letting students lead any conversation/ discussion or any problem solving activity. As a result of such "enthusiasm" for leading the conversation, George was not able to create dialogical interaction among students. Such cases shows that George fluctuated between "letting go" and taking the control of lesson to catch up with the curriculum. This resulted in taking less account of students' problem solving processes and more of having them do mechanistic parts of mathematics as in his control classroom.

Another promising conclusion deriving from this study is that understanding of and finding a solution for students' misconceptions might be related to teachers' subject matter knowledge and pedagogical knowledge. At the beginning, George called on John about his suggestion for the solution. Even though John offered the wrong method, George decided to follow John's ideas. This instructional decision made by George helped him to understand John's thinking process about the problem and to realize what was wrong. On the other hand, George could not interpret students' ideas in the light of his understanding of mathematics; that is, in terms of the big ideas of the lesson he identified, George could not emphasize the importance of the use of real numbers, which resulted in a possibility that might have led students to think that only whole numbers could be used as the dimensions of a rectangle. George's unattentiveness to students' misconceptions was because of his dominance in classroom discourse and problem solving process.

There were some limitations in the study. First of all, this study was about a single teacher's lesson. The same analysis can be done by observing several teachers to outline how teachers' flexibilities create learning opportunities for students. Secondly, this snapshot of George's lesson was out of a series of already videotaped lessons. Therefore, beforehand, the teacher was not guided to plan for such a goal. Another study can be conducted where teachers' lesson plans were documented and their real implementations were analyzed according to those plans (Leikin & Kawass, 2005). The variation can be scrutinized in detail.

References

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Borasi, R. (1992). *Learning mathematics through inquiry*. Portsmouth, NH: Heinemann Educational Books.

Brendefur, J. & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher*

Education, 3, 125-153.

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education*, 27, 174-186.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal of Research in Mathematics Education*, 28, 258–277.

Cuban, L. (1993). *How teachers taught: Constancy and change in American classrooms, 1890-1990* (2nd ed.). New York, NY: Teachers College Press.

Edwards, D. & Mercer, N. (1987). *Common knowledge: The growth of understanding in the classroom*. London: Routledge.

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics.* Albany, NY: State of University of New York Press.

Esterberg, K.G. (2002). *Qualitative methods in social research*. McGraw-Hill, Boston.

Gunel, M. (2006). *Investigating the impact of teachers' practices of inquiry and non-traditional writing on students' academic achievement of science during longitudinal professional development program.* Unpublished PhD Dissertation, Iowa State University, Ames.

Hoekstra, A., Brekelmans, M., Beijaard, D., & Korthagen, F. (2009). Experienced teachers' informal learning: Learning activities and changes in behavior and cognition. *Teaching and Teacher Education*, 25, 663–673.

Johnson, E.M.S. & Larsen, S.P. (2012). Teacher listening: The role of knowledge of content and students. *Journal of Mathematical Behavior, 31*, 117-129.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven: Yale University Press.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education 61(1-2),* 21-34.

Leikin, R. & Dinur, S. (2007). Teacher flexibility in mathematical discussion. *Journal of Mathematical Behavior, 26*, 328-347.

Leikin, R. & Kawass, S. (2005). Planning teaching an unfamiliar mathematics problem: The role of teachers' experience in solving the problem and watching pupils solving it. *Journal of Mathematical Behavior, 24,* 253-274.

Martin, A. M., & Hand, B. (2009). Factors affecting the implementation of argument in the elementary science classroom. A longitudinal case study. *Research in Science Education, 39*, 17-38.

McMillan, J. & Schumacher, S. (1997). *Research in education: A conceptual introduction*. (4th ed.). New York: Addison Wesley Longman, Inc.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Obara, S. & Sloan, M. (2009). Classroom experiences with new curriculum materials during the implementation of performance standards in mathematics: A case study of teachers coping with change. *International Journal of Science and Mathematics Education, 8*, 349-372.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20(2)*, 119-150.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(4), 4-14.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26(2),* 114-145.

Simon, M. A. (1997). Developing new models of mathematics teaching: An imperative for research on mathematics teacher development. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 55-86). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Treagust, D. (2007). General instructional methods and strategies. In S. Abell, & N. Lederman (Eds.) *Handbook on research in science education* (pp. 373–391). Mahwah, NJ: Lawrence Erlbaum Associates.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge: Harvard University Press.

Walshaw, M. & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78* (3), 516-551.

Wilson, M. & Goldenberg, M. P. (1998). Some conceptions are difficult to change: One middle school mathematics teacher's struggle. *Journal of Mathematics Teacher Education, 1*, 269-293.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *Journal of Mathematical Behavior, 21*, 423-440.