Sixth Grade Students' Performance on Length, Area, and Volume Measurement

Altıncı Sınıf Öğrencilerinin Uzunluk, Alan ve Hacim Ölçülerindeki Performansları

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Abstract

The purpose of this study was to investigate sixth grade students' conceptual and procedural knowledge and word problem solving skills in the domain of length, area, and volume measurement. Through Conceptual Knowledge, Procedural Knowledge, and Word-Problem Tests, data was collected from 445 sixth-grade students attending public primary schools in Ankara, Turkey. The findings revealed that the students were more successful in computational tasks than tasks requiring acquisition and coordination of underpinnings of measurement. A significant relationship not only between the tests, but also between the domains of measurement was observed. The overall performances of the students on the tests significantly differed according to previous mathematics achievement, but no gender difference was observed.

Keywords: Conceptual and procedural knowledge, word problem solving, length, area, and volume measurement

Öz

Bu çalışmanın amacı, altıncı sınıf öğrencilerinin uzunluk, alan ve hacim ölçüleri konularındaki kavramsal ve işlemsel bilgilerini ve sözel problem çözme becerilerini araştırmaktır. Çalışmanın verileri, Ankara İli'ndeki devlet ilköğretim okullarında öğrenim gören 445 altıncı sınıf öğrencisinden Kavramsal Bilgi Testi, İşlemsel Bilgi Testi ve Sözel Problem Testi yoluyla toplanmıştır. Bulgular, ölçüler konusunun temelini oluşturan kavramların anlamlandırılması ve koordinasyonunu gerektiren sorulara göre, öğrencilerin işlemsel bilgiye dayalı sorularda daha başarılı olduğunu göstermiştir. Bunun yanı sıra, öğrencilerin hem testlerdeki başarısı arasında hem de ölçüler konusunun alt boyutları (uzunluk, alan ve hacim) arasında anlamlı bir ilişki bulunmuştur. Ayrıca, öğrencilerin testlerde gösterdikleri genel başarı, önceki döneme ait matematik dersi genel başarı notuna göre anlamlı olarak farklılaşırken, cinsiyet açısından anlamlı bir fark bulunamamıştır.

Anahtar Sözcükler: Kavramsal ve İşlemsel Bilgi, Sözel Problem Çözme, Uzunluk, Alan ve Hacim Ölçme.

Introduction

Measurement is an essential part of daily life and holds a unique part in mathematics programs as well as in other subject areas (Baroody, & Coslick, 1998; Pope, 1994). It is one of the universal activities for the growth of mathematical ideas and focuses on comparing, ordering, and quantifying qualities (Bishop, 1988; as cited in Kordaki & Portani, 1998). Hart (1984) explained the importance of measurement as a mathematical strand by suggesting that "If teachers of

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mathematics were asked to choose the five or six most important topics in the school mathematics curriculum, then measurement would be likely to appear on every list" (p.16). Considering its fundamental role in mathematics, science, and our life, it is obvious that students should fully understand not only "how to measure" but also 'what it means to measure".

Several research studies on teaching and learning measurement has indicated that students have difficulties in understanding the concepts and skills of measurement (Kloosterman, et al., 2004; Martin & Strutchens, 2000). Most educators in this field agree that the reason behind students' poor understanding of measurement is due to putting more emphasis on how to measure rather than to what to measure means for them (Grant & Kline, 2003; Kamii & Clark, 1997). In this respect, knowing mathematical concepts and procedures as well as the relationship between them are the fundamental elements for meaningful mathematical learning.

Kilpatrick, et al., (2001) defined conceptual knowledge as "comprehension of mathematical concepts, operations, and relations" (p. 5). Students with conceptual knowledge understand what a mathematical concept is, why it is important, and the contexts in which it is suitable (Kilpatrick, et al., 2001). On the other hand, Hiebert and Lefevre (1986) characterized procedural knowledge as knowledge of mathematical algorithms or procedures that are "step-by-step instructions that prescribe how to complete tasks" (p.6). Knowledge of procedures enables students to use formulas, algorithms and symbols appropriately and flexibly (Kilpatrick, et al., 2001). From the perspective of conceptual knowledge, measurement is full of various kinds of concepts (e.g. unit iteration, perimeter, surface area, volume, etc.). It is also one of the strands that includes rich connections within its own content (e.g. understanding of area concept provides foundation for the concept of volume) and within others (e.g. the underlying principle behind both the ruler and the number line). From the perspective of procedural knowledge, measurement is also rich in terms of skills (e.g. using tools to measure length, area, volume) and formulas (e.g. for finding perimeter, area, volume).

It is obvious that not only knowledge of mathematical concepts but also knowledge of mathematical procedures are the gatekeepers for learning mathematics with understanding, and as a result, becoming an effective problem solver. As stated by the National Council of Teachers of Mathematics (2000), the main reason for teaching mathematics is to teach how to solve problems. Silver (1986) pointed out that mathematical problems are important vehicles for the development of both conceptual and procedural knowledge, as problem solving process entails making use of both types of knowledge. Among mathematical tasks, word problems have continued to be a special part of almost all mathematics curricula and textbooks. A word problem is defined as "verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement" (Verschaffel, Greer, & De Corte, 2000, p. xi). Word problems are vital for promoting students' mathematical understanding in terms of connecting different meanings, interpretations, and relationships with mathematics operations (Van de Walle, 2007) and they create a context for the development of new concepts and skills if they are wisely selected and sequenced (Verschaffel, et al., 2000). In the light of the above mentioned ideas, the main purpose of this study was to investigate sixth grade students' conceptual and procedural knowledge and word problem solving skills in length, area, and volume measurement with respect to the selected variables (gender and previous mathematics achievement). More specifically, the study focused on the determination of differences in students' performances when three domains of measurement (length, area, and volume) were assessed by different tests (conceptual, procedural, and word problems) and the examination of the differences and relationships among the selected variables (gender and previous mathematics achievement).

Although past studies have shed light on students' thinking about measurement, conducting a research study on students' conceptual and procedural knowledge and word-problem solving skills in length, area, and volume measurement, which are off-cited problematic areas, is valuable from several perspectives. First of all, to our knowledge, there is no research study looking into students' conceptual and procedural knowledge as well as word-problem solving skills in length, area, and

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volume measurement together. Previous studies on measurement generally examined students' (a) success in a set of measurement tasks, (b) developmental levels, and (c) strategies related to only a one domain of measurement (e.g. length). In this respect, the whole picture of students' understanding of measurement including the knowledge of concepts and procedures as well as the skill for solving word-problems on length, area, and volume measurement may not be drawn completely. However, knowing to what extent students make sense of length, area, and volume measurement and to what extent they are capable of applying the knowledge of measurement in these three domains would enable teachers and curriculum developers to design effective measurement instruction. Further, Turkey has made substantial changes in the elementary school curricula in that elementary mathematics curriculum as well as other subject areas were redeveloped and implemented throughout the country at the start of the 2005-06 academic year. At this point, determining students' current knowledge of measurement may also provide helpful insights for teaching of measurement concepts and skills.

When compared to the number of studies on measurement conducted abroad it can be observed that research on teaching and learning measurement is still an emerging area of study in Turkey. Emekli's study (2001) which aimed to diagnose 7th and 8th grade students' misconceptions in measurement indicated that a majority of students had misconceptions about the use of fractions in measurement, conservation of area, formulas for perimeter, area and volume, and estimation of measurement. Olkun's (2003) study with 4th through 7th grade students' performance and strategies for finding the number of unit cubes in rectangular solids revealed that students' success increased and their strategies became more complex from 4th grade to 7th grade. In their study of the investigation of fifth grade students' understanding of measurement, Albayrak, Isik, and Ipek (2006) reported that most of the students distinguished the measurable and non-measurable attributes of objects (e.g. the width of a book and eye color) and chose the appropriate unit for the attribute being measured, yet their performances were quite low on the tasks related to unit conversions and expressing measures in terms of another standard units. The results of the Tan-Sisman and Aksu's study (2009) indicated that a majority of the 7th graders had serious problems with the concept of area and perimeter, some misconceptions, and had difficulties in using the formulas for area and/or perimeter effectively. In this respect, it is believed that the results of this study may contribute not only the related literature by giving an example from the Turkish educational context but also lays the foundation for future research on measurement for the mathematics education community in Turkey.

Finally, the results of the present study might offer significant suggestions and guidelines both for mathematics teachers and curriculum developers who would like to develop students' conceptual and procedural knowledge as well as their word-problem solving abilities.

Research questions

The present study essentially sought to answer the following questions:

- **1.** What are the students' overall performances on conceptual knowledge, procedural knowledge, and word problem tests?
- **2.** Is there any significant relationship among the students' performance on these three tests?
- **3.** Do the students' performances on these three tests differ according to gender and previous mathematics achievement (5th grade mathematics report card score)?

Method

Participants

A sample of 445 sixth grade students was selected from six different public primary schools in Ankara, Turkey. The sample selection was based on the schools' average mathematics scores in the National Selection Examination for Secondary Education Institutions (OKS) which was a highly-competitive nationwide examination administered at the end of eighth grade, especially for those students who would like to enroll into one of the well-resourced, qualified and prestigious high schools. Additionally, the performances of the schools in the OKS exam were perceived by parents as a strong indicator of the quality of education provided in primary schools (Sahin, 2004).

According to official records of The Ministry of National Education, the total number of public primary schools in Ankara that participated in the OKS was 685 and the average mathematics score of these schools was 1.43 out 25. Based on the range, the minimum, and the maximum OKS scores, the public primary schools in Ankara were classified as low-, medium-, and high-achieving schools and two schools from each achievement level was selected for the study. All sixth grade students attending the selected public schools constituted the participants of this study. The detailed information about the sample is presented in the table 1.

Table 1.

Achievement levels	Selected schools	Average Math Scores in the OKS 2006	Central Districts	Number of 6 th graders participated in the study
High	X School Y School	7.27 6.65	Cankaya Cankaya	81 81
Medium	Z School	4.51	Yenimahalle	83
	F School	3.31	Kecioren	67
Low	L School	0.59	Kecioren	50
	K School	-0.58	Altindag	83

Distribution of the Schools and the Students Selected for the Study

Among the 445 sixth grade students whom participated in the study, 203 of them were male and 242 were female. Their ages ranged between 11-14 and most of them (N=387, 87%) were 12 years old. In relation to the students' previous mathematics achievement (mathematics report card grade in 5th grade), a majority of them (N=377, 84.7%) were high-achievers.

Data Collection Instruments

In the present study, Conceptual Knowledge Test (CKT), Procedural Knowledge Test (PKT), and Word Problems Test (WPT), were used as main data collection instruments. Through synthesizing the information gathered from the literature, content and the learning objectives of length, area, volume measurement topics in the sixth grade mathematics curriculum, three tests were developed by the authors [only two of the questions in CKT were taken from the different sources, one of which was an adapted version of TIMSS-1999 released item (Permanent ID M022168) and another taken from Hart's study (1981)]. Since the present study focused on students' performances related to the fundamental concepts and skills of spatial measurement; all tests were developed in line with the learning objectives of length, area, and volume measurement in the Turkish National Mathematics Curriculum from first to sixth grades. Seventh and eighth grades were not considered due to the fact that the learning objectives become more specific and detailed in terms of the geometrical shapes at these grade levels (e.g. to use formulas to find the areas of rhombus, parallelograms, trapezoids, and circles). The following table provides detailed information about the data collection instruments of the present study.

Table 2.

General Information about the Data Collection Instruments

	СКТ	РКТ	WPT
Number of tasks	16	20	20
Number of sub-tasks	50	-	-
Completion time	40-45 min.	35-40 min.	40-45 min.
Maximum Score	50	20	20

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The CKT was designed to examine to what extent students comprehend the conceptual underpinnings of the three domains of measurement, and thus, the items neither asked students to determine the correct/incorrect answers among the alternatives nor required to carry out computational exercises. Instead, constructed-response format tasks asked students to show understanding of measurement concepts by interpreting, applying, and transferring them correctly to different situations. The sample question from the CKT is given below.



Figure 1. The Sample Question in the CKT

Another data collection instrument of the study was the PKT, designed to investigate the extent to which students could apply procedures (routine and complex). For this reason, the test only involved the tasks triggering application of the skills related to length, area, and volume measurement, such as calculations, rules, formulae, etc. The sample question from the PKT is given below.



Figure 2. The Sample Question in the PKT

The last data collection instrument was the WPT developed to assess the students' word problem solving skills in three domains of measurement. It was composed of both routine and non-routine problems written in the form of verbal statements and each of them involved the same numbers and operations with the questions in the PKT. In other words, each word problem has a pair in the PKT which was presented in the numerical form and had the same numbers and operations needed to reach the solution. The sample question from the WPT is given in the following figure.

The Sample Question from the Word Problem Solving Test Measurement Domain: Area Skill Embedded in the Question: To solve a word problem involving the area of a rectangle Jane's father wants to cover her room which is 5 meters long and 4 meters wide with new carpet. What is the amount of carpet her father will need to buy? Show all your work.

In order to obtain content-related and face validity evidences, the tests were given to three academicians from the field of mathematics education, two academicians from the field of educational sciences and three mathematics teachers. In addition, a pilot-study was conducted with 7th grade students (N = 134) who already received the instruction on measurement during sixth grade, but were not taught the content of 7th grade measurement. The internal consistency of the three tests was obtained through the KR-20 formula. The reliability coefficient was found as .87 for the CKT, .88 for the PKT, and .89 for WPT indicating high internal consistency (Fraenkel & Wallen, 2001).

Procedure

Upon completing the regular instruction on the length, area, and volume measurement in the schools, the tests were administered to sixth grade students in three different sessions on different days, since administering all instruments in one session would be too long and tiring for students. A scoring key for each test was prepared by the researchers and also, revised by the two academicians from the field of mathematics education and a mathematics teacher. Each scoring key involved possible correct and incorrect answers for each task and subtasks in the tests and 1 point was assigned for the correct answer and 0 for both the incorrect answer and blank question.

Since the administration of the tests was done on different days, some of the students missed some of the sessions. Therefore, those who were not present during the administration of all 3 tests were excluded from the data analysis process. Descriptive and inferential statistical methods were then carried out by a means of Predictive Analytics Software (PASW) in order to answer the questions raised in the current study.

Results

Students' Overall Performances on the Tests

The first research question aimed to investigate the overall performance of sixth grade students' in three tests. As indicated in table 3, the overall scores of the students in each test were quite low. Considering the students' performance on three domains of measurement, the highest performance was observed in length measurement and the lowest in volume measurement in each test.

Table 3.

Students'	Performance				
Overall	-	М	SD	Min.	Max.
CKT	(out of 50 points)	19.6	9.2	0	46
PKT	(out of 20 points)	8.3	4.7	0	20
WPT	(out of 20 points)	7.7	4.8	0	20
Length M	easurement	М	SD	Min.	Max.
CKT	(out of 24 points)	12.2	4.6	0	24
PKT	(out of 7 points)	4.7	1.7	0	7
WPT	(out of 7 points)	4.4	2	0	7
Area Mea	surement	М	SD	Min.	Max.
CKT	(out of 15 points)	5	3.3	0	15
PKT	(out of 8 points)	2.1	1.9	0	8
WPT	(out of 8 points)	1.7	1.8	0	8
Volume N	leasurement	M	SD	Min.	Max.
CKT	(out of 11 points)	2.4	2.5	0	11
PKT	(out of 5 points)	1.5	1.7	0	5
WPT	(out of 5 points)	1.5	1.7	0	5

Students' Performance on the CKT, PKT, and WPT (n=445)

Indeed, among the linear measurement tasks of the CKT, the sixth graders got the lowest score (M = .04, SD = .21) in the question presented in the following figure.

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Figure A

Figure B

A square-shaped paper, as shown in Figure A, is folded in the middle and is cut in half. Then, one of the pieces is cut again as shown above. Figure B is formed by putting the pieces together as shown in the picture. Based on the information given, choose one of three comments about the figures and explain your reasoning.

Comment 1: The perimeter of Figure A is greater than the perimeter of Figure B.				
🗆 I agree, because				
Comment 2: The perimeters of Figure A and Figure B are equal.				
🗆 I agree, because				
Comment 3: The perimeter of Figure B is greater than the perimeter of Figure A.				
🗆 I agree, because				

Figure 4. The Lowest Performance Length Measurement Question in the CKT

On the other hand, the highest performance (M_{CKT} = .83, SD_{CKT} = .37) was observed in the task asking

"what is the most appropriate unit of measure for the distance between two cities". In both the PKT and WPT, the students showed the highest performance (M_{PKT} = .86, SD_{PKT} =.34; M_{WPT} = .77, SD_{WPT} = .45) on the task asking to find the perimeter of a square as presented in the figure 5.



Figure 5. The Highest Performance Length Measurement Question in the PKT and WPT

Further, in the length measurement task regarding conversion from kilometers to meters, students got the lowest score in the PKT and WPT ($M_{PKT} = .53$, $SD_{PKT} = .49$; $M_{WPT} = .51$, $SD_{WPT} = .50$). The lowest and highest performances on length measurement tasks in each test are given in chart 1.





Considering area measurement, students' well-performed area measurement task was choosing the most appropriate unit of measure for the area of football yard in the CKT ($M_{CKT} = .41, SD_{CKT} = .49$). Nevertheless, the task asking sixth graders to explain what would happen to the surface area, if the volume of a cube is halved was the lowest performance area task in the CKT (M = .03, SD = .17). In both the PKT and WPT, the students demonstrated the highest performance on the task related to calculating the area of a rectangle ($M_{PKT} = .64, SD_{PKT} = .47; M_{WPT} = .54, SD_{WPT} = .50$); the lowest performance ($M_{PKT} = .02, SD_{PKT} = .15; M_{WPT} = .02, SD_{WPT} = .16$) was observed in the task asking to find the height of a square prism, where its side length and the surface area were provided. The highest and lowest performance area measurement tasks according to the tests are presented in the chart 2.



Chart 2. The Highest and Lowest Performance Area Measurement Tasks by Tests

In relation to the volume measurement, sixth graders showed the highest performance on the CKT task (M = .51, SD = .50) which asked them to choose the most appropriate unit of volume measurement for the amount of water in a swimming pool. With a 6.5% correct response rate (M = .06, SD = .24), the lowest performance was observed in the CKT's task presented below.



Figure 6: The Lowest Performance Volume Measurement Task in the CKT

Like in length and area measurement tasks, the highest and lowest performance on volume measurement questions in PKT and WPT were also observed in the parallel questions. While the students got the highest score in the task related to making conversion from dm³ to m³ (M_{PKT} = .40, SD_{PKT} = .49; M_{WPT} = .38, SD_{WPT} = .48), the lowest performance was observed (M_{PKT} = .27, SD_{PKT} = .44; M_{WPT} = .23, SD_{WPT} = .42) in the task asking to find the height of a rectangular prism where the volume and two dimensions were given. The highest and lowest performance volume measurement tasks according to the tests are provided in the chart 3.





The second research problem was asked to find whether there was any significant relationship among the sixth grade students' overall performances on these three tests. Bivariate correlations was computed among the three tests and the Pearson correlation coefficient values, given in table 4, indicated that there was statistically significant and strong relationships ($r_{pKT-WPT}$ =.84, p < 0.5; $r_{CKT-WPT}$ =.73, p < 0.5; $r_{CKT-PKT}$ =. 70 p < 0.5). In other words, the results clearly revealed a strong and positive interrelationship between understanding of the measurement concepts, carrying out operations with measurement, and solving word problems involving measurement. Therefore, an increase in one test type (e.g. CKT) might lead to gains in other tests (e.g. PKT and/or WPT) and vice versa.

Table 4.

Correlatio	n Matrix	for	the	CKT	PKT	and	WPT
Corretatio	11 1111111	101	ine	CKI	$1 \Lambda 1$	ипи	V V I I

Overall Score	СКТ	PKT	WPT
СКТ	1.00		
PKT	.70*	1.00	
WPT	.73*	.84*	1.00
Length Measurement	СКТ	PKT	WPT
CKT	1.00		
РКТ	.56*	1.00	
WPT	.59*	.71*	1.00
Area Measurement	СКТ	PKT	WPT
CKT	1.00		
РКТ	.51*	1.00	
WPT	.54*	.75*	1.00
Volume Measurement	CKT	РКТ	WPT
СКТ	1.00		
РКТ	.61*	1.00	
WPT	.61*	.82*	1.00
*p<.05			

Similarly, bivariate correlation was also run in order to see whether there was a significant relationship among the sixth graders performance on the tests with regard to domains of measurement. The results yielded a significant relationship, positive and strong, both between

and within students' performance on the tests in terms of three domains of measurement. Put differently, when the students' performance in one domain of measurement (e.g. area measurement) increased in one of the tests (e.g.PKT), their performance on the same domain of measurement in the other tests (e.g.CKT) increased, too.

Students' Overall Performance on the Tests by Gender and Previous Mathematics Achievement

The last research problem aimed to explore whether the students' overall performance on the tests differ according to gender and previous mathematics achievement. For this purpose, a multivariate analysis of variance (MANOVA) was conducted by using PASW. The result of Pillai's Trace Test was reported in this study so as to yield robust statistic against unequal sample sizes (Field, 2009). Table 5 illustrates F-statistics for both multivariate and univariate analysis.

Table 5.

Multivaritate and Univariate Analysis of Variance: CKT, PKT, and WPT by Gender and Previous Mathematics Achievement

		ANOVA		
	MANOVA	СКТ	РКТ	WPT
Gender	V=.01, F(3, 437)	F(1,439)=.87	F(1,439)=.63	F(1,439)=.009
PMA	V= .10, F(6, 876)*	F(2,439)=14.46**	$F(2,439)=22.10^{**}$	F(2,439)=21.90**

Note. F ratios are Pillai's Trace approximation. **p*<.05, ***p*<.017 (Bonferroni- type adjustment) PMA: Previous Math Achievement

As seen in table 5, the findings revealed that there were significant differences in the students' overall performances on the combination of three dependent variables: CKT, PKT, and WPT by previous mathematics achievement V = .104, F(6,876) = 8, p < .05, $\eta^2 = .052$ indicating a small effect. Namely, about 5% of the variance in the combined dependent variables (CKT, PKT, and WPT) is explained by previous mathematics achievement. Nonetheless, no significant difference was observed in the students' performance on the tests in terms of gender, V = .008, F(3,437) = 1.22, p > .05.

Discussion and Conclusion

The present study mainly aimed to investigate sixth grade students' conceptual and procedural knowledge and word problem solving skills in length, area, and volume measurement in terms of differences and relationships. The findings revealed that the students performed quite poorly in all tests. Indeed, although the sixth graders were relatively successful in routine or textbook-type questions (e.g. making conversions within metric system, calculating the perimeter of a square), they had serious difficulties in the tasks requiring more than execution of operations (e.g. explanation of the relationship between surface area and volume, the structure of a ruler). Furthermore, the lowest mean scores were observed in the WPT, then CKT, and PKT respectively. Thus, it might be concluded that the students had quite limited knowledge about "what measurement means" and "how to measure" and consequently, had difficulties in solving word problems involving measurement. These results corroborate the ideas of Baroody, et al., (2007), Hiebert and Lefevre, (1986), Rittle-Johnson, et al., (2001), Silver, (1986) etc., who claimed that students' mathematical competence is mostly built on both the knowledge of concepts and procedures in a mathematical domain and thus, with the help of knowing what/why and how to do, students can make sense of mathematics and effectively use their repertoire of conceptual and procedural knowledge in problem solving situations. With regard to domains of measurement, decreasing performance was observed from one-dimensional (length), to two-dimensional (area), and to three-dimensional (volume including surface area). The findings might be a consequence

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of the Turkish Elementary Mathematics Curriculum where length measurement is taught and practiced in 1st grade, area measurement in 3rd grade, and volume measurement in 5th grade. As a result, students might have had more opportunities to develop the concepts and skills involved in length measurement, than area and volume measurement (Tan-Sisman & Aksu, 2012). Nevertheless, when compared to the total scores of the tests, the mean score of length, area, and volume measurement in each test was actually low. In this respect, it might be argued that neither six-year-study of length measurement at school is effective for students to gain underlying concepts and procedures of measurement.

One of the more significant findings to emerge from this study was that there was a strong and positive interrelationship among understanding of the measurement concepts, carrying out operations with measurement, and solving word problems involving measurement. Rittle-Johnson and her colleagues' (2001) study produced similar results. According to the results of their experimental study with fifth and sixth graders, the relationship between conceptual and procedural knowledge was bidirectional and conceptual knowledge had power to develop procedural knowledge and vice versa. Star's (2002) research study also found positive relationship between knowledge of concepts and procedures in the case of equation solving. It was additionally observed that the more students know about, for instance, length measurement, the more likely they are to be successful in the other domains of measurement. Like other mathematical strands, measurement has special principles that are composed of the concepts and procedures underlying and justifying measurement process. Without making sense of these principles unique to each domain of measurement, it is extremely difficult for students to learn to both do and understand measurement (Stephan & Clements, 2003; Lehrer, 2003; Kamii & Clark, 1997). In this respect, the findings of this study confirmed again the significant and positive relationship among the concepts, procedures, and word-problem solving skills in each domain of measurement.

Moreover, results also indicated that girls and boys had nearly the same mean scores in each test. This result differed from what Lubienski (2003) found. In the study he reported, measurement was the only content area in which the largest gender differences was observed since 1990 in the previous National Assessment of Educational Progress exams. However, as claimed by several researchers (Leder, 1985; Peterson & Fennema, 1985; as cited in Alkhateeb, 2001), during the elementary school years, the gap between boys and girls mathematics achievement was not obvious and clear. One of the more noteworthy findings to emerge from the present study is that the previous year's mathematics grade (5th grade) was significantly related to the performance on all three tests. As the students' prior mathematics achievement increased, their performance on the tests increased, too. These results are consistent with Kabiri and Kiamanesh's (2004) findings. Additionally, Bragg and Outhred (2000), and Battista (2003) underlined that students' understanding of length measurement is crucial for understanding of area and volume measurement. Thus, these results support the view that students' prior achievement in mathematics is one of the strong predictors for their subsequent success in mathematics. Indeed, these results might also be interpreted as an indicator of the cumulative and sequential nature of mathematics.

The overall results of the study revealed superficial and inadequate understanding and skills of the sixth graders in length, area, and volume measurement which is obviously not the intended and desired learning outcome of the mathematics curriculum. In this respect, the findings might be considered as evidences calling for comprehensive research on the causes of the poor performance of students and for the curricular and instructional changes in measurement strand. First and foremost, the foundational concepts and skills of length, area, and volume measurement and the relationship among them should explicitly be taken into consideration both by curriculum developers, teacher educators, and teachers themselves. Especially, zeropoint, unit iteration, structure of a ruler, relationship between the attribute being measured and a unit of measurement being used, understanding that perimeter might be changed under

partitioning, difference between perimeter and area and between their formulas, spatial structure, multiplicative structure, and conservation of area, spatial visualization, meaningful enumeration of arrays of cubes, the difference between surface area and volume and between their formulas should become integral part of measurement strand.

Through challenging and mathematically-rich tasks, special care should be taken to ensure that students comprehend the underlying principles of measurement and its domains. For instance, although students can measure the length of a string and find that 12 centimeters is the correct answer, they do not understand what "12" signifies. In this respect, teachers should provide students with plenty of learning opportunities that promote meaningful understanding of measurement before introducing formulas and tools. Moreover, unless we know for sure what students understand and think about measurement, we fail to design effective measurement instruction (Stephan & Mendiola, 2003). Providing teachers with research-based explicit knowledge about student's thinking in a specific content domain positively affects teachers' instruction and students' achievement (Carpenter, et al., 1989). At this point, it might be suggested that this kind of research-based information should be included in mathematics curriculum to inform teachers about difficulties students' face in measurement. Besides, prospective teachers should be equipped with the areas of students' deficiency about measurement during their teacher education process. Considering the essentiality of measurement in science, technology, mathematics, and everyday life, unless we carefully design and implement well-planned highquality mathematics curriculum in which the study of measurement begins from students' naïve ideas (e.g. building arrays of units) to gradually continue with more sophisticated ideas (e.g. how the length and width produce an area, as a result of multiplication) that lead to real understanding, we will probably fail to create mathematically literate societies for the 21st century.

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