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Pre-service Mathematics Teachers' Reflections on Cognitively Demanding Tasks and the Characteristics of Algebra Tasks They Developed *

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Abstract

This study aims to investigate pre-service mathematics teachers' (PMTs) reflections on the 2018-2020 High School Entrance (HSE) exam questions and the characteristics of the algebra tasks developed by the PMTs after participating in an intervention that involves examining and categorizing algebra questions. It also examines the degree to which PMT-generated tasks are cognitively demanding and consistent with the characteristics of HSE exam questions. A case study was employed in this study as a qualitative research design. The study was conducted in the 2020-2021 Spring semester as part of the Methods of Teaching Mathematics in Middle Schools II course offered in the 3rd year of a 4-year teacher education program in a public university. A total of 29 PMTs enrolled in the course, and they were asked to work in groups. The study intervention took place during the course, and one of these groups was focused on during the intervention. The data were collected through two algebra tasks that the group produced at the end of the intervention, four PMTs' individual responses to a written form, and their semi-structured individual interviews about reflections on HSE exam questions. All the collected data were qualitatively analyzed using content analysis. Both existing categories in the literature and new emergent categories were used in this process. Four categories emerged from PMTs' reflections on HSE exam algebra questions: algebra objectives, use of algebra, use of context, and cognitive demand after participating in the intervention. The findings about the characteristics of the tasks developed by PMTs indicated that they were capable of developing cognitively demanding algebra tasks, which were mostly consistent with PMTs' reflections on the characteristics of HSE exam questions. The implications of the findings for PMTs' reflections on HSE exam questions and the tasks they developed were also discussed.

Keywords

Mathematics education
Pre-service mathematics teachers
Cognitively demanding tasks
High School Entrance (HSE) Exam
Algebra

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Introduction

One of the central aims of education is to empower students with the skills to think critically, solve problems, and transfer their skills to new situations. To achieve this goal, schools must provide opportunities for students to engage in tasks requiring these higher-order thinking skills. Mathematical tasks are a particularly effective way to promote the development of such skills as they require students to think about, apply, and connect mathematical concepts (National Council of Teachers of Mathematics [NCTM], 1991). The value of employing such tasks with students lies in the learning opportunities they provide, such as enabling students to make connections, share their insights, and deepen their understanding of mathematics (Webb, 2009).

Mathematical tasks can be used for many purposes such as instruction and assessment. Despite their different purposes, they all serve the main purpose of teaching and learning since teachers who strive to improve student understanding are also required to assess it (Webb, 2009). The tasks that mathematics teachers select, adapt, develop, and implement in the classroom are of paramount importance since they influence the degree of student learning and convey to the learners what mathematics is and what doing mathematics involves (NCTM, 1991). Teachers' mathematical task knowledge for teaching, in particular, can be the determining factor in teachers' usage of tasks in this respect (Chapman, 2013). Chapman (2013) described the mathematical task knowledge for teaching as the knowledge teachers require to select and develop tasks that help students understand mathematical concepts, develop mathematical thinking, and stay engaged. Teachers also need this knowledge to optimize the learning potential that these tasks afford. Teachers' task knowledge involves many dimensions, including "understanding the nature of worthwhile tasks," "knowledge of levels of cognitive demands of tasks," and "the ability to identify, select and develop mathematically and pedagogically rich tasks" (Chapman, 2013, p. 1). In addition, it requires putting these skills into practice.

Students' mathematics practice activities may vary depending on the types of tasks in which they engage (Shimizu, Kaur, Huang, & Clarke, 2010). The tasks can provide different contexts that engage students in the subject matter and activate lower or higher-order thinking skills (Doyle, 1983; Hiebert & Wearne, 1993). Students' higher-order thinking skills are activated when they work on unfamiliar or challenging tasks (King, Goodson, & Rohani, 2018) that require explanations, interpretations, justifications, generalizations, or decision-making. Students need to analyze these tasks and explore the nature of mathematical concepts, relationships, and processes involved in these tasks. These actions require considerable cognitive effort (Smith & Stein, 1998). Students who engage in higher-order thinking skills can learn how to enhance their learning and eliminate their learning deficiencies (Tanujaya, 2016).

Teachers are responsible for selecting or developing mathematical tasks to enhance students' understanding. Hence, they need to be aware of the complexity of decision-making in the process of task selection and development to use in the classroom. The investigation into the skills of teachers in developing mathematical tasks revealed that both in-service and pre-service teachers may lack the ability to produce sufficient mathematical tasks. Preservice mathematics teachers rely on the internet or textbooks and use readily available tasks in their classroom, believing that writing tasks is "someone else's business" (Ellerton, 2013, p. 88). Pre-service teachers also feel unprepared to create their original tasks (Mallart, Font, & Diez, 2018) and experience difficulty designing them (Silver, Mamona-Downs, & Leung, 1996). When pre-service teachers were asked to create tasks, their products were found to be poorly formulated, predictable, undemanding, or unsolvable (e.g., Crespo, 2003; Silver et al., 1996). These indicate that in-service and pre-service mathematics teachers need to possess the professional competence to develop tasks that activate and measure students' high-level thinking skills. Mathematical task knowledge is quite demanding because it requires knowledge of complex structures and constructing mathematical task knowledge is challenging for teachers without meaningful intervention (Chapman, 2013). Thus, teacher education programs should support in-service and pre-service mathematics teachers in this respect.

Research revealed the results of successful interventions and challenges for guiding in-service and pre-service teachers to improve their mathematical task knowledge. Some studies indicated that engaging pre-service and/or in-service teachers with analyzing, adapting, or developing mathematical tasks enhanced their task writing skills (e.g., Carson, 2010; Crespo, 2003; Kaplan-Can, 2023; Leavy & Hourigan, 2020; Norton & Kastberg, 2012; Prestage & Perks, 2007). Some also suggested that in-service or pre-service teachers improved their understanding of quality mathematical tasks and/or cognitive demands by analyzing, revising, and implementing cognitively demanding tasks through professional development workshops or teacher education sequences (e.g., Boston, 2013; Kaplan-Can, 2023). This means that in-service and pre-service teachers' task knowledge can be enhanced through well-designed interventions. The present study aims to help PMTs develop cognitively demanding algebra tasks as a result of an intervention in the context of HSE examination.

Use of Cognitively Demanding Tasks in High School Entrance (HSE) Exam

The tasks that help teachers activate students' higher-order thinking skills have gained prominence in the Turkish education system with the change in the HSE examination system in 2018 (Kertil, Gülbağcı-Dede, & Ulusoy, 2021). The HSE exam is one of the large-scale high-stakes tests used to place 8th-grade students in high schools; therefore, it is quite important for them. Several studies (e.g., Biber, Abdulkadir, Uysal, & Kabuklu, 2018; Kablan & Bozkuş, 2021; Kertil et al., 2021) pointed out that the structure of the items in the HSE exam was different compared to previous exams (e.g., Level Determination Examination, Transition to Secondary Education from Basic Education). Before the new HSE examination system was implemented, the mathematics questions in previous exams administered with the same purpose measured basic skills at the level of knowledge, comprehension, and application (Başol, Balgalmış, Karlı, & Öz, 2016; Yakalı, 2016). Now, the questions aim to measure high-level skills such as making interpretations, drawing inferences, and analytical thinking (Biber et al., 2018). The questions require mathematical thinking and interpretation beyond just using the given information. Furthermore, unlike the previous exam questions, the HSE exam questions are more contextual and related to daily life (Biber et al., 2018; Kablan & Bozkuş, 2021). These questions also generally contain visuals such as tables, graphs, and images related to daily life (Kablan & Bozkuş, 2021). In addition, the questions may involve multiple mathematical ideas and allow for more than one solution. These types of tasks are often called "new generation questions" by most teachers or "skill-based questions" by some scholars (Kertil et al., 2021, p. 152). These questions used to be unfamiliar to students and teachers because teachers generally refrain from utilizing such questions during their traditional mathematics teaching process before the new HSE examination system (Kertil et al., 2021).

Reports informing the HSE exam results from 2018 to 2022 revealed that the mathematics subtest scores consistently exhibited the lowest average number of correct responses compared to all other subtests (Ministry of National Education [MoNE], 2018a, 2019, 2020, 2021, 2022). This result suggests that students might still be unprepared for cognitively demanding skill-based tasks, and mathematics teachers may lack the necessary competence to foster students' higher-order thinking skills. Thus, it becomes imperative for mathematics teachers to comprehensively understand higher-order thinking skills and strategies to enhance them.

The research findings indicated that teachers had little access to the necessary resources to use in their classrooms (e.g., Biber et al., 2018). The amount of resources teachers can use in their lessons is relatively more sufficient than in previous years because many sample questions are published for teachers by the MoNE. On the other hand, teachers' and students' primary resources in the classrooms are textbooks, and teachers usually organize the mathematics courses through the mathematical tasks and activities in the textbooks (Lepik, Grevholm, & Viholainen, 2015). However, teachers think that mathematics textbooks are not enough sources to meet their needs for the HSE exam because the cognitive demands of the questions in the textbooks are not compatible with the HSE exam questions (Obay, Demir, & Pesen, 2021). Furthermore, even though teachers rely on extra supplementary textbooks published by private publishers, they often criticize the quality of the questions in those sources concerning clarity, context, and the required operations to solve them (Yılmaz & Şad, 2022). Consequently, the significance of teachers developing their original tasks becomes increasingly noticeable.

Based on the literature and the assumption that the HSE exam questions are cognitively demanding, this study focuses on PMTs' reflections on HSE exam questions (2018-2020). Various studies have recently been conducted on the new HSE examination system. However, the studies on the HSE exams generally focused on investigating teachers' opinions about the HSE exam questions (Biber et al., 2018; Erden, 2020; Güler, Arslan, & Çelik, 2019; Kablan & Bozkuş, 2021) or classification of mathematics test questions according to Bloom's revised taxonomy (Ekinci & Bal, 2019; Üzümcü & İpek, 2022; Yılmaz & Doğan, 2022). Kablan and Bozkuş (2021) found that according to teachers' opinions, HSE mathematics exam questions required high-level thinking skills such as reasoning, analyzing, and interpreting. Furthermore, HSE mathematics exam questions provided a context related to daily life, and they were also difficult to understand because their texts were long. Ekinci and Bal (2019) investigated the level of cognitive demand of HSE 2018's mathematics exam questions according to Bloom's revised taxonomy. They found that the cognitive levels of HSE mathematics exam questions were at *apply* and *analyze* levels. Similarly, Yılmaz and Doğan (2022) examined the mathematics exam questions of the HSE exam administered in 2021 according to Bloom's revised taxonomy. Their findings revealed that the levels of cognitive demand of these questions were at *apply, analyze, and evaluate* levels.

As Kertil et al.'s (2021) findings revealed, teachers need professional development to write quality skill-based questions and develop problem-solving skills. This necessitates studies investigating teachers' reflections on HSE exam mathematics questions' characteristics and whether they can generate cognitively demanding tasks to enhance in-service and pre-service mathematics teachers' professional development. Accordingly, the purpose of this study is to examine PMTs' reflections on the cognitively demanding questions and the characteristics of the algebra tasks developed by the PMTs after participating in an intervention involving the examination and categorization of algebra questions used in the 2018-2020 HSE examinations. This study also aims to understand whether pre-service teachers can generate cognitively demanding tasks and whether these tasks are consistent with HSE exams' question characteristics.

Therefore, the research questions guiding this study are as follows:

1. What are the reflections of PMTs on the cognitively demanding questions after participating in an intervention involving examination and categorization of algebra questions used in the 2018-2020 HSE examinations?
2. What are the characteristics of the algebra tasks developed by the PMTs after participating in an intervention involving examination and categorization of algebra questions used in the 2018-2020 HSE examinations?
3. To what extent were PMTs' reflections connected to the algebra tasks they developed after participating in an intervention involving examination and categorization of algebra questions used in the 2018-2020 HSE examinations?

Method

In this study, we investigated the characteristics of the algebra tasks developed by the PMTs according to Bloom's revised taxonomy (Anderson & Krathwohl, 2001) and Smith and Stein's (1998) categorizations of cognitive demand. Furthermore, we focused on the algebra learning area as part of the Methods of Teaching Mathematics in Middle Schools II course.

Algebra is often characterized as a crucial gateway to higher mathematics courses (Stacey & Chick, 2004), and it constitutes a fundamental part of the math curriculum globally (Leung, Clarke, Holton, & Park, 2014). Algebra is also found to be closely related to other learning areas, particularly geometry and data analysis (NCTM, 2000) and is viewed as the language of mathematics (Grønmo, 2018). Considering the crucial role of algebra in school mathematics and the focus of algebra in the method course content, we specifically focused on PMTs' examination and development of cognitively demanding tasks in the algebra learning area in this study.

Research Design

This study employed the case study design, a qualitative research design. Yin (2009) emphasizes that the case study fully investigates a contemporary phenomenon in its real-life context, and the investigator has little or no control over the events. The case in this study comprised the selected group of four PMTs' reflections on the cognitively demanding questions and the characteristics of algebra tasks that they developed during the intervention.

Participants and the Context of the Study

The study was conducted as part of the Methods of Teaching Mathematics in Middle Schools II course offered in the 3rd year of a 4-year teacher education program in a public university. This teacher education program aimed to train teachers for middle schools (grades 5 to 8). In the program's first two years, PMTs are offered mostly mathematics and introductory education courses. In their third year, primarily mathematics education courses start to be offered. They are offered three methods of teaching courses, each focusing on different learning areas. In the program's last year, they are offered Practice Teaching courses. This study was conducted in the 2020-2021 Spring semester (PMTs' sixth semester).

Before the Methods of Teaching Mathematics in Middle Schools II course, PMTs were introduced to Smith and Stein's (1998) categorization of the cognitive demand of mathematical tasks briefly in a course offered during their fourth semester. Also, in their fifth semester, they were enrolled in the Assessment of Learning in Science and Mathematics course, in which they were introduced to different types of assessment and writing multiple-choice, true/false, closed-, and open-ended items at differing cognitive levels utilizing Bloom's revised taxonomy (Anderson & Krathwohl, 2001). Hence, PMTs were familiar with Smith and Stein's (1998) cognitive level categorizations and Bloom's revised taxonomy before the study. The categorizations and the taxonomy are explained in the data analysis section.

Methods of Teaching Mathematics in Middle Schools II course, delivered by one of the research team members, focused on proportional thinking, algebraic thinking, data, and probability as content. The course spanned 14 weeks and consisted of two 2-hour sessions per week (see Appendix 1). In total, 29 PMTs enrolled in the course. Elementary and Middle School Mathematics: Teaching Developmentally by Van de Walle, Karp, and Bay-Williams (2013) was the main book of the course, and some other practitioner journal articles were also shared with PMTs. The class was usually run as a combination of whole-class and small group discussions. The sessions were held over an online video conferencing software "Zoom", due to the emergency distance education. At the end of the algebraic thinking sessions, two groups, which had been determined at the beginning of the semester, implemented their lesson plans related to the chapter of the coursebook through microteaching. Then, the study intervention was administered during the course.

The intervention spanned five 2-hour sessions (see Figure 1 and Appendix 1). In the first session, all the PMTs enrolled in the course were introduced and invited to the study. They were also introduced to their group project assignment, where they were asked to submit two open-ended cognitively demanding algebra tasks. Then in the following three sessions, before the class, the PMTs were asked to examine the HSE exam questions individually used in 2018, 2019, and 2020, respectively, and fill out a table that instructed them to classify algebra questions, explain why they can be classified as algebra, whether there was a context in each, what objective(s) each addressed, and determine the cognitive levels of the tasks according to the Bloom's revised taxonomy and Smith and Stein's (1998) categorization with their reasons (see Appendix 2). During the class sessions, the PMTs were asked to work in groups. The groups (6 groups in total) were created heterogeneously based on their GPAs. During the sessions, the PMTs were asked to review and finalize their tables with their group members. The same groups worked together throughout the intervention. In this study, students in one of these groups, Group 1, were selected because they were willing to participate in the study and the course instructor believed that they would be willing to openly share their ideas and dedicate time for interviews. This group included 4 PMTs (three female and one male). Toward the end of the sessions, whole-class discussions were held around some past HSE exam questions from the ones they reviewed that week. These discussions focused on the classification they generated regarding those selected questions, such as why or why not the groups classified the items as algebra, whether there was a context or not, and why, as well as the level of the questions' cognitive demand. In the fifth and final session, all groups presented the two open-ended algebra tasks they were asked to prepare and submit. The PMTs were asked to develop open-ended tasks to prevent limiting them by thinking of tasks only for HSE exam questions. They were also asked to present their reasoning for why these two tasks were algebra tasks, the related objectives from the curriculum, cognitive levels according to Bloom's revised taxonomy and Smith and Stein's (1998) categorization for each task, and justifications for the categorizations.

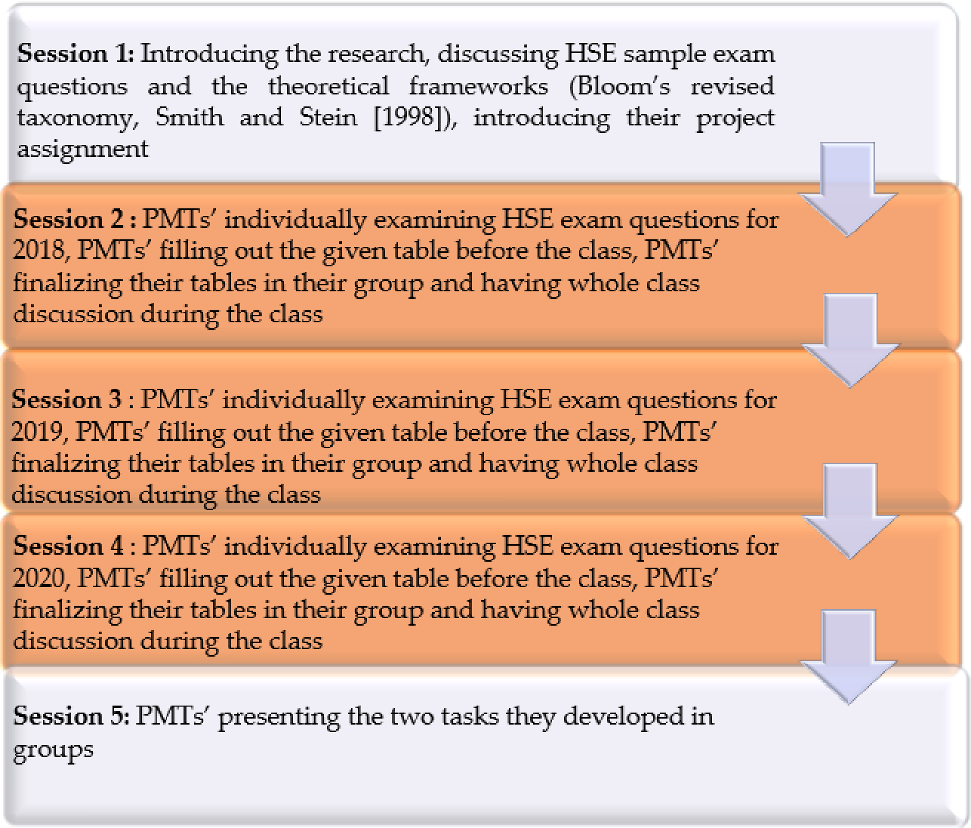


Figure 1. The intervention process

Data Collection

The data sources of this study were Group 1's (selected group) two open-ended algebra tasks (see Figures 2 and 3), including PMTs' group reasoning about the tasks, four PMTs' individual responses to a written form, and their semi-structured individual interviews about reflections on HSE exam questions.

After the intervention summarized in Figure 1 was implemented, each group of PMTs developed two cognitively demanding open-ended tasks and provided their reasoning on the task characteristics. A written form was also collected from all PMTs enrolled in the course at the end of the intervention. This form included questions asked to understand in what ways the intervention was helpful for them. These questions were: 1. Did examining HSE exam questions contribute to you? Explain your answer (positive or negative). 2. Did the process of developing algebra questions targeting higher-order thinking skills contribute to you? Explain your answer (positive or negative). The questions asked were intended to be written in general to prevent influencing them or limiting their reflections.

Semi-structured individual interviews were conducted with four PMTs in Group 1 via Zoom and recorded with participants' permission. These interviews lasted between 60 and 100 minutes. The interview protocol consisted of questions about the intervention and their task development processes. For instance, PMTs were asked "You examined HSE 2018, 2019, and 2020 exam questions during the intervention process. What do you think about the structure of the HSE exam questions? How would you describe the questions?" to understand their thoughts about the structure of HSE exam questions. Another example question was the following: "What are the procedures you followed while developing two cognitively demanding algebra questions?" This question was asked to reveal the criteria they considered in developing their tasks. Some other questions were also asked to uncover their responses to the written form, such as "You expressed that analyzing HSE exam questions contributed to you in "........." aspect(s) in the written form. In addition to this, do you think there were other contributions?" was an example of those questions.

Data Analysis

PMTs' transcribed interviews and written forms were read and coded using content analysis. Both existing categories in the literature and new emergent categories derived from the data were used in this process (Fraenkel, Wallen, & Hyun, 2012).

The data analyses were conducted in three phases. In the first phase, the data obtained on PMTs' reflections regarding the HSE exam questions were coded. PMTs' reflections on the HSE exam algebra questions included four emergent categories: algebra objectives, the use of algebra, the use of context, and cognitive demand. In the second phase, the characteristics of the tasks that PMTs developed were analyzed. As summarized in Table 1, the codes for cognitive demand coming from Bloom's revised taxonomy (Anderson & Krathwohl, 2001) and Smith and Stein's (1998) categorizations were utilized to analyze the levels of the tasks that PMTs developed.

Table 1. The code list used to categorize the characteristics of tasks PMTs developed

Theme	Categories	Codes
The characteristics of the tasks that PMTs developed	Cognitive Level according to Taxonomies (Anderson & Krathwohl, 2001; Smith & Stein, 1998)	Bloom's Revised Taxonomy • remember, • understand, • apply, • analyze, • evaluate, • create. Smith and Stein (1998) • memorization, • procedures without connection (PW/oC), • procedures with connection (PWC), • doing mathematics.
	Context (De Lange, 1995)	Relevant and Essential Camouflage No context
	Core Aspects of Algebra (Kaput, 2008)	Core Aspect A Core Aspect B

Bloom's revised taxonomy classifies thinking into six cognitive levels of complexity: remember, understand, apply, analyze, evaluate, and create (Anderson & Krathwohl, 2001). The first two or three categories (remember, understand, and sometimes apply) represent lower-level thinking skills, and the last three or four categories (analyze, synthesis/create, and evaluate and sometimes apply) represent higher-level thinking skills (e.g., McDavitt, 1994; Norton & Rutledge, 2010). The first category, *remember*, is defined as recalling or recollecting ideas, symbols, theorems, or rules. At this level, students are expected to remember the information presented in the lesson or in a textbook and express it in a similar way (Birgin, 2016). The second category, *understand*, involves constructing the meaning of instructional messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining (Krathwohl, 2002). Students can transform knowledge into a new form, such as a graphic, table, or figure, and explain the related concepts by interpreting and summarizing the phenomenon (Baki, 2008). The third category, *apply*, may be considered low-level (McDavitt, 1994) or high-level (Norton & Rutledge, 2010), depending on the context. It includes carrying out a procedure by executing or implementing, and students apply what they know in a new situation at this level. Questions in the fourth category, *analyze*, ask students to break the material into its parts and describe how parts relate to each other and the overall purpose or structure through differentiating, organizing, and attributing (Birgin, 2016; Krathwohl, 2002). At this level, students can reach generalizations, make connections between various fields, and establish a cause-effect relationship (Birgin, 2016). The fifth category, *evaluate*, requires making a judgment based on a criterion by checking and critiquing. The last category, *create*, requires forming a coherent or original whole through generating, planning, and producing (Krathwohl, 2002). Questions at these levels may require students to compare and contrast new ideas or theories or generate their original ideas. Students can evaluate a unique body of knowledge and its product from various perspectives and develop original solutions for problems (Birgin, 2016).

Similarly, Smith and Stein (1998) leveled the cognitive demand into four categories: (i) memorization, (ii) procedures without connection (PW/oC), (iii) procedures with connection (PWC), and (iv) doing mathematics. They suggested these categories to help teachers select, adopt, or design cognitively demanding mathematical tasks that increase students' ability in analyzing and reasoning. Similar to Bloom's revised taxonomy, Smith and Stein (1998) identified the first two categories as low-level demands and the last two categories as high-level demands. Memorization, the first level of the lower demands, involves recalling previously learned rules, facts, formulas, or definitions. Such tasks are not ambiguous and involve reproducing previously seen material (Smith & Stein, 1998). The tasks in the second lower level, PW/oC, are algorithmic. They require limited cognitive effort since what needs to be done and how to do it are clearly stated. They also do not involve a connection to the other concepts. The tasks in the first level of the higher demands, PWC, require some cognitive effort. Students engage with conceptual ideas to develop an understanding and complete the task successfully. These tasks are usually represented in several ways, such as manipulatives, symbols, and diagrams (Smith & Stein, 1998). The tasks in the highest level of cognitive demand, doing mathematics, require complex and non-algorithmic thinking. Students need to understand and explore the nature of mathematical concepts or relationships to solve such tasks. The solution process of these tasks is generally unpredictable, so students examine the task to limit the solution strategies (Smith & Stein, 1998).

The context of the tasks was categorized based on De Lange (1995). De Lange (1995) discriminated the use of contexts into three categories: "relevant and essential," "camouflage," and "no context." Relevant and essential contexts require engaging in relevant mathematics to organize and solve the task. Understanding and using context is necessary to solve a task where context is both relevant and essential. The camouflage context, on the other hand, does not affect solving the task. Tasks without context are mathematical problems not embedded in real or camouflage contexts.

The reasons for categorizing a task as an algebra task were coded using Kaput's (2008) framework. Kaput defined two core aspects of algebra, which are a) "algebra as systematically symbolizing generalizations of regularities and constraints", and b) "algebra as syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems." (p. 11). Students are encouraged to notice regularities and form generalizations related to Core Aspect A, while rule-based actions are more prevalent in Core Aspect B.

In the third phase, to analyze to what extent PMTs' reflections were observed in the algebra tasks they developed, PMTs' reflections on the cognitively demanding questions were compared with the characteristics in the developed algebra tasks.

Validity and Reliability

Inter-coder reliability was used to increase the reliability of the coding (O'Connor & Joffe, 2020). Specifically, each piece of data obtained from different data sources was coded independently by at least two research team members. The characteristics of the tasks developed by Group 1 considering the categories of cognitive demand, context, and core aspects of algebra were separately coded by three research team members independently, and the agreement among coders was 100% in the first round of the coding process. Furthermore, the written form and interview data were coded by two of the research team members independently. The inter-coder reliability across the data sources was calculated using Cohen's Kappa (Cohen, 1960). Cohen's Kappa values for the written form data and the interview data were 0.89 and 0.92, respectively. So, the initial agreement between coders was 89% for the written form data and 92% for the interview data, indicating a high-level agreement between coders. Disagreements that were caused by the codes of cognitive demand category and use of context category were discussed until all team members reached 100% agreement.

To ensure validity, triangulation, specifically data source triangulation was used in the study (Creswell, 2007). Multiple data sources, the tasks developed, individual written forms, and semi-structured individual interviews, were used in the study in the data collection to focus on PMTs' reflections on the cognitively demanding questions and the characteristics of algebra tasks they developed during the intervention. Furthermore, rich and thick descriptions of the research context and participants were provided (Creswell, 2007).

Results

In this part, the findings will be presented in the order of the research questions. The first question focused on PMTs' reflection on the HSE exam questions, and the second one investigated the characteristics of the algebra tasks developed by the PMTs after participating in an intervention. The third question investigated the connection between PMTs' reflections on HSE exam questions and developed algebra tasks.

PMTs' Reflections on HSE Exam Questions

After the intervention, reflections of four PMTs in Group 1 on HSE exam questions were sought in a written form; their individual responses were followed up in the post-interview conducted individually. The question in the written form was whether examining HSE exam questions was helpful for them and to explain why. In response to this question, PMTs mainly indicated their reflections on HSE exam items. Four major categories emerged: algebra objectives, the use of algebra, the use of context, and cognitive demand. These are detailed in Table 2.

Table 2. PMTs' Reflections on HSE exam algebra questions

Categories	PMTs' reflections on HSE exam algebra questions
Algebra Objectives	• The frequency of some algebra objectives that were addressed • Having a better knowledge of the algebra learning area objectives • The HSE exam questions' covering not only 8th-grade's objectives but those of the previous years as well[1] • The HSE exam questions' measuring multiple learning area objectives
The Use of Algebra	• How algebra is related to other learning areas[2] • How algebra is used in the HSE exam questions[3]
The Use of Context	• The use of context in the HSE algebra exam questions[4] • How context use changed in years from 2018 HSE exam questions to 2020 HSE exam questions • Being able to use context in developing questions
Cognitive Demand	• The level of cognitive demands of the HSE exam questions • How the questions should be written so that they are at the specified level[5] • Being able to classify HSE exam algebra questions according to the specified taxonomies

Note1: These categories are not necessarily mutually exclusive.

Note2: The superscript numbers (1, 2, 3, 4, and 5) written for PMTs' reflections indicate the connections between their reflections on HSE exam questions and the characteristics of developed algebra tasks. See the last subtitle in the results section, "Making Connections Between PMTs' Reflections on HSE Exam Questions and Developed Algebra Tasks by PMTs."

Under the algebra objectives category, PMTs mentioned what they noticed related to the curriculum algebra objectives. For instance, related to the sub-category, "HSE exam questions covering not only 8th-grade's objectives but those of the previous years as well," PMT4 said, "I discovered that the questions were not only used to measure a specific objective but also to measure the cumulative mathematical knowledge that had been built up to that point." Related to the "the use of algebra" category, PMT3 stated, "Does it [the question] require her [the student] to establish equations in general, or does it require her to find different values in algebraic expressions..." Thus, PMT3 stated his reflections on the use of algebra in the HSE exam questions. Related to "the use of context" category, PMT2 noted that in 2019 and 2020, the questions were more contextual than in 2018. Lastly, related to "cognitive demand" category, PMT3 stated his reflection as follows. Here, the PMT3 reflected on how the question should be written so that it can be categorized as *analyze* level.

> *...how we write, [the question, so] children can analyze. It helped me understand them. When we look at the old questions… The child just needs to apply what he has learned earlier in a new situation. But in such questions, ... [the child needs] to think first, to decide something cognitively, to judge and decide what to do instead of solving it directly.*

In the post-interviews, the PMTs were also asked about the structure of the HSE exam questions and how they would define the questions. These categories were similar to those found in the previous question, where they were asked about their reflections after examining HSE exam questions. The major categories that emerged were that the questions were contextual or relevant to daily life, they required higher-level thinking, and they focused on more than one learning area. All PMTs stated that questions included a context and a daily life component. For instance, PMT2 stated, "…for example, the integration of slope questions into daily life… here is where we will use it. There is actually an answer to that. You can use the slope here." Likewise, PMT3 said, "It is nice to use it in daily life, so children can at least think about that question and think about where it works for us or where it can be encountered in daily life." The other major category was the questions requiring higher-level thinking. In this category, PMT4 noted, "It requires students not only to memorize but also to think a bit. Even I had a hard time solving some questions." PMT2 similarly added, "We were memorizing directly… But right now, it [the question] is already giving it to us. It doesn't want us to memorize it, but it asks us to use it. I think this would be more helpful." The last major category related to the structure of the HSE exam questions was the questions focusing on more than one learning area. Related to this category, PMT1 explained, "I like that it covers a few different topics... that is, several different topics can be combined in questions." PMT4 stated, "They tried to combine algebra with slightly different areas of mathematics. They used geometry. It was pretty interesting when we think about it in that context. Its structure has changed over the years." As shown in PMTs' quotations, similar to their reflections in the written form, they emphasized context, cognitive demand and focusing on more than one learning area in the post-interviews.

The Characteristics of the Tasks Developed by PMTs

As stated earlier, this study focused on Group 1. In the post-interviews, Group 1 emphasized that they did not use any resources while writing their tasks. The characteristics of the tasks as described by PMTs are presented in Table 3.

Table 3. The Characteristics of the Tasks Developed by Group 1

Tasks	Is there a context?	The level of cognitive demand according to Smith and Stein (1998)	The level of cognitive demand according to Bloom's Revised Taxonomy	Why is this task an algebra task?
T1	Yes	PWC	Analyze	"This is an algebra task since we have a few variables, and the value of these variables are changing [from] case to case."
T2	Yes	PWC	Analyze	"This is an algebra task since there are unknown values of the sides, and these sides can only be found with the equation that we make with slope."

In the following sections, the characteristics of each task will be introduced in detail.

The characteristics of Task 1

The first task (T1, see Figure 2) developed by Group 1 had multiple parts, as presented below. In Part a, students are required to calculate the calories that Yuji and Sakura could take daily to maintain weight without gaining any. In this part, students are expected to substitute the given values into the equation and solve it for different values. In Part b, students need to decide whether Yuji is taking enough calories per day to maintain his weight based on his daily diet plan. This means that students are expected to compare the daily calorie intake of Yuji with his Basal Metabolic Rate (BMR) to decide whether it is enough per day. In Part c, students are required to decide on how to increase/decrease the portions that Yuji consumes if his calorie intake is higher/lower. This means that students need to propose a daily diet that leads them to make calculations by increasing/decreasing the portions with the numbers provided.

Task 1

Dieticians are professionals who help people to form healthy eating habits. They use some mathematical calculations to determine the number of calories that could be taken by a person. The calorie calculation is based on the basal metabolic rate (BMR) and activity level of a person. You can think of BMR as a required energy for the body to continue functioning. Activity level refers to the daily activities of a person, and we will use it to estimate the calorie intake.

The formula for required calorie intake is $BMR \times Activity\ Level$

Activity Level 1	Activity Level 2	Activity Level 3	Activity Level 4
$BMR \times 1.2$	$BMR \times 1.375$	$BMR \times 1.725$	$BMR \times 1.9$

The formula of BMR is:

<u>For female:</u> BMR=$(10 \times Weight) + (6.25 \times Height) - (5 \times Age) - 161$

<u>For male:</u> BMR= $(10 \times Weight) + (6.25 \times Height) - (5 \times Age) + 5$

You are given the information of two people.

Name	Yuji	Sakura
Gender	Male	Female
Age	20	28
Height(cm)	179	156
Weight(kg)	75	60
Activity level	4	2

a. By using the information above, calculate the calories that Yuji and Sakura could take daily to maintain their weight without gaining any. ***(It is important not to take in more calories to maintain the current weight.)***

b. Decide whether Yuji is taking enough calories per day to maintain his weight based on his daily diet plan which is given below. Explain your decision.

Breakfast	3 whole scrambled eggs	304 calories
	1 ½ cups plain yogurt with ½ cup blueberries	150 calories + 42.5 calories
	1/3 cup granola	157 calories
Snack	Filter coffee with milk (1 portion)	78 calories
Lunch	Hamburger (1 and ½ portions)	531 calories
	Mediterranean salad (1 portion)	142 calories
Snack	Green tea ice cream (1 portion)	250 calories
Dinner	Grilled chicken (2 portions)	452 calories
	Quinoa salad (1 portion)	296 calories
	Coke (1 can)	150 calories

c. Based on the answer you found in Part b, comment on what Yuji can do to balance his diet? How can you increase/decrease the portions Yuji consumes? You can make an estimation.

Figure 2. The First Task Developed by Group 1

PMTs expressed that "this is an algebra task since they have a few variables, and the values of these variables are changing [from] case to case." The PMTs focused on the change in variables in their justification. They also expressed that their task had a context that could be relevant to everyone. PMT3 explained the group's reasoning in the post-interview as follows:

PMT3: That is food calories… It is important for everyone, even for those who are underweight, overweight, and those who are at such ideal weight. They need to calculate how much they need to eat to maintain their ideal weight.

According to them, their tasks required high-level thinking. The PMTs' decision about the level of cognitive demand of Task 1 was PWC, according to Smith and Stein's (1998) categorization. In their group project assignment where they presented their tasks, they justified why the level of the task was PWC as follows:

Even though they [students] are familiar with algebraic expressions and solving algebraic equations, this task requires some degree of cognitive effort since it includes more than one variable. They [students] have to think about how they can solve the problem they encounter. Also, the other two parts of the task demand some decision-making and evaluation processes, so they cannot mindlessly follow some general procedures.

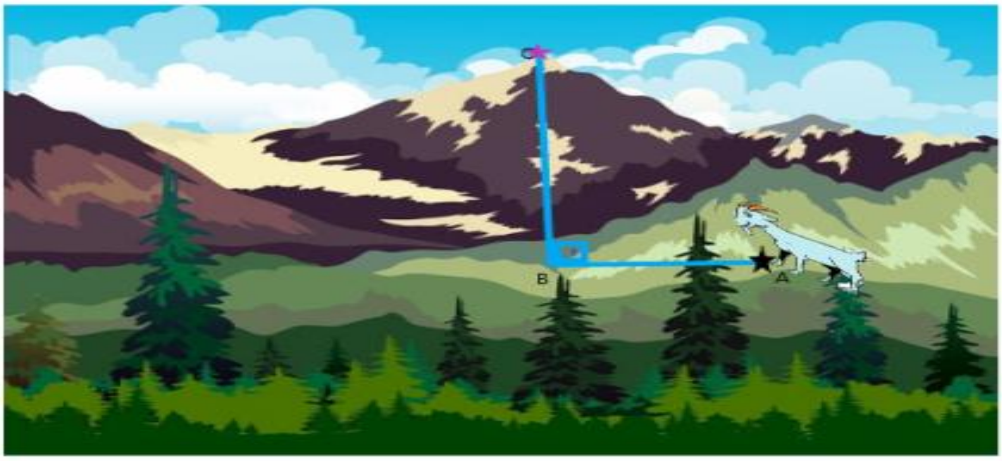
On the other hand, in their project, PMTs identified the overall level of their task as *analyze*. In the post-interview, they were asked how they decided on the cognitive demand of the task and whether they distinguished the level of parts. PMT3 and PMT4 stated that they asked some students to solve the task and realized that students need to think about and cannot solve the task easily. In addition, PMT1 expressed that since it requires comparison, it is a high-level task. PMT2 and PMT4 also emphasized that they discussed the levels of cognitive demands of the parts. They concluded that the level of Part a is *apply,* Part b is *analyze,* and Part c is *evaluate*. However, they decided to report the overall level of the task as *analyze*.

The characteristics of Task 2

The second task (T2; see Figure 3) asks students to find the shortest distance that a goat can use to climb to the top of the mountain using a right angle. Students are expected to write the short sides' lengths in terms of x and find the value x using the slope given in the task. They are expected to form a Pythagorean relation to find the shortest distance.

Task 2*

Goats are experts in climbing steep slopes. Even the most competent climbers need safety equipment such as ropes, rock nails, hammers, and seat belts during rock climbing. Mountain goats, on the other hand, can climb even on steep slopes without equipment.



Above is given a path through which a mountain goat can climb to the top with a slope value of 2. The distance between point B and point C, the top of the mountain, is 200 less than 3 times the distance between point A, where the goat is located, and point B. What is the smallest value the $|AB|$ path can take for the goat to reach the summit by the shortest route? (You may use the calculator.)

*Group 1 stated that they adapted the image from a website for the task.

Figure 3. The Second Task Developed by Group 1

The PMTs in Group 1 stated that their task was an algebra task because the side lengths of the right triangle can be found with the equation formed using the slope. They indicated that the task had a context. While expressing the relevancy of the context of their tasks, PMT3 stressed that "Everyone can visualize since there is a picture, and it is the kind of thing one might encounter while traveling, and you know, people can often come across goats climbing a mountain in documentaries." In addition, they classified its level of cognitive demand as PWC, considering Smith and Stein's (1998) categorization. The following excerpt from the project assignment displayed the PMTs' justification for their decision: "The path the children will follow is not clearly stated. It is not clear what the children should do. So, it requires some cognitive effort. In addition, a connection has been established with geometry."

The PMTs in Group 1 also expressed that its level of cognitive demand was *analyze* since the "task required a combination of information from different mathematical areas. Students should use this information to solve the problems."

Analysis of the Tasks Developed by the PMTs

The analysis of the tasks by the researchers indicated that most of the characteristics of their tasks were consistent with the properties of cognitively demanding algebra tasks. The task characteristics analyzed by the researchers using the frameworks based on Bloom's revised taxonomy and Smith and Stein's (1998) for cognitive level categorizations, the categories for the context of the tasks based on De Lange (1995), and lastly, the categorization for algebra (Kaput, 2008) are presented in Table 4.

Table 4. The Characteristics of the Tasks According to The Researchers' Analysis

Tasks	Context	The level of cognitive demand according to Smith and Stein (1998)	The level of cognitive demand according to Bloom's Revised Taxonomy	Why is this task an algebra task?
T1	Relevant and Essential	PWC	a. Apply b. Apply c. Analyze	Core Aspect B
T2	Camouflage	PWC	Apply	Core Aspect A

The data analysis revealed that T1 was an algebra task because the task required calculating a formula using variables with different values. The task focused on students' knowledge in the "algebra" and "numbers" learning areas (MoNE, 2018b). Furthermore, researchers decided that T1 was a contextual algebra task that was relevant and essential (De Lange, 1995). That is, the context of the task was necessary to understand and solve the task. Also, consistent with the PMTs' decision, the researchers determined the task's level of cognitive demand as PWC according to Smith and Stein's (1998) categorization. In this task, students are required to engage with conceptual ideas, including using the information from the table provided and deciding on the daily diet by increasing/decreasing the meal portions and explaining their reasoning. However, contrary to the PMTs' decision, the researchers determined each part's level of cognitive demand separately. Considering Bloom's revised taxonomy, Part a and Part b were categorized at the *apply* level since, in Part a, students are expected to substitute the given values into the equation and calculate it, while in Part b, they need to add all the calories in the table and compare the result with the one they found in Part a. Moreover, Part c was categorized at *analyze* level since students are required to propose a daily diet by increasing/decreasing the portions. T1 was categorized under Core Aspect B according to Kaput (2008), since the task focused on calculating a formula by using variables with different values. In other words, it required the manipulation of symbolism.

Similar to T1, T2 was an algebra task, according to researchers, because the task required setting up/formulating an equation (using Pythagorean relations) containing an unknown that represents problem situations. Group 1 used the geometric ratio ($slope = rise/run$) meaning of the slope in line with the objective M.8.2.2.6 in the middle school mathematics curriculum (MoNE, 2018b). T2 focused on using "algebra" and "geometry and measurement" learning areas (MoNE, 2018b). The researchers decided that T2 was contextual and had a camouflage context (De Lange, 1995). That is, the context of this task could be neglected while solving the task, and students could follow the procedures easily without thinking about the context. The researchers also decided that the problem context of T2 lacked clarity because the path PMTs referred to in the text was not clearly indicated, and the unit of the given distance (200) was missing. The lack of clarity of the context affected task quality. However, as the PMTs expressed, the path was not stated in the task intentionally, and students were expected to interpret the relation between the shortest route, the path which the goat could climb and the slope value of the path. That is, because students would not be able to proceed directly to the solution of the task without engaging with the idea of slope to formulate the equation, the solution of the task involved some degree of cognitive effort. Therefore, consistent with the group's decision, the researchers decided that the task's level of cognitive demand was PWC in accordance with Smith and Stein's (1998) categorization. However, the researchers' categorization of the level of T2 differed from the group's decision according to Bloom's revised taxonomy level because combining different learning areas does not guarantee that the task was at the *analyze* level. Since students were required to use the formula (Pythagorean relation) they learned before in a new problem situation, its level of cognitive demand was categorized as *apply* by the researchers. T2, where the students were expected to set up an equation using the Pythagorean theorem, was categorized under Core Aspect A according to Kaput (2008), since it required formulating an equation containing an unknown that represents a problem situation.

Making Connections between PMTs' Reflections on HSE Exam Questions and Developed Algebra Tasks by PMTs

When PMTs' reflections on the cognitively demanding questions after participating in an intervention were compared with the characteristics in the developed algebra tasks, it was observed that they used five of their reflections while developing them (see Table 2 and Note2). PMTs reflected that the HSE exam questions covered not only 8th-grade objectives but also the ones of the previous years[1]. Accordingly, in their project, PMTs reported that the objective of their first task (T1) was at the 6th-grade level (MoNE, 2018b, M.6.2.1.2*), and the objective of their second task (T2) was at the 8th-grade level according to the middle school mathematics curriculum (MoNE, 2018b, M.8.2.2.6). PMTs also indicated that the HSE exam questions focused on more than one learning area, and they used two learning areas in both tasks[2]. Furthermore, another reflection of PMTs was about how algebra was used in the HSE exam algebra questions. Categorizing their tasks using Kaput's (2008) framework, both core aspects were found to be addressed by their tasks, namely Core Aspects A and B[3]. In addition, PMTs noticed the use of context in the HSE algebra exam questions[4]. In line with their reflections, both tasks were contextual. Lastly, PMTs' reflections about how the questions should be written so that they are at the specified level[5] were observed in their tasks. Although the level of cognitive demand of their tasks was different according to Bloom's revised taxonomy, both algebra tasks were PWC according to Smith and Stein's (1998) categorization. Accordingly, they required high-level thinking and cognitive effort.

Discussion, Conclusions, and Suggestions

The study examined individual reflections of four PMTs working in a group on the 2018-2020 HSE exam questions, the characteristics of tasks PMTs developed as a group, and the connections between their reflections on HSE exam questions and the algebra tasks they developed after participating in an intervention involving examination and categorization of algebra questions in the previous HSE examinations.

The study's findings initially revealed that PMTs in Group 1 reflected on the fundamental characteristics of HSE exam questions after participating in an intervention. Their individual reflections on HSE exam questions were grouped under four main categories: algebra objectives, the use of algebra, the use of context, and cognitive demand. PMTs' reflections aligned with their teachers' opinions as revealed in other studies (e.g., Biber et al., 2018; Erden, 2020; Güler et al., 2019; Kablan & Bozkuş, 2021). In line with the teachers' opinions reported in previous research, PMTs in this study noticed that HSE exam questions were mostly contextual or relevant to daily life and required higher-level thinking. As different from the previous research findings, though, PMTs in the current study noticed that the HSE exam questions covered not only 8th-grade objectives but also those of the previous years. They also recognized that the questions focused on more than one content; that is, the algebra questions were related to other learning areas, particularly geometry.

The findings on the characteristics of the tasks developed by PMTs in Group 1 indicated that most of the characteristics of their tasks were consistent with the properties of cognitively demanding algebra tasks. According to the researchers' analysis, both tasks involved higher-order thinking. One of the tasks (T1) was at the *analyze* level, and the other (T2) was at the *apply* level according to Bloom's revised taxonomy. Furthermore, the tasks' level of cognitive demand was PWC according to Smith and Stein's (1998) framework. Both tasks also required interpretation and problem-solving beyond just using the information in the context.

*The group specified that M.6.2.1.2 was not a relevant objective for their task during the interview because the objective focused on algebraic expressions, not equations. Still, they decided to use it as they found it the most relevant one in the curriculum.

Mathematical task knowledge, as indicated, is a complex domain that requires meaningful intervention to enhance (Chapman, 2013). The promising results of successful interventions such as engaging teachers in analyzing, adapting, and developing mathematical tasks by participating in workshops or teacher education sessions present valuable pathways for improving in-service or pre-service teachers' task-writing skills and deepening their understanding of the level of cognitive demand (e.g., Boston, 2013; Kaplan-Can, 2023). In line with this idea, the study's design might have helped PMTs reflect on the many characteristics of HSE exam questions and develop cognitively demanding tasks. For instance, PMTs' examination of HSE exam questions' cognitive levels according to Bloom's revised taxonomy and Smith and Stein's (1998) framework could provide a rich learning environment to understand the level of cognitive demand of the questions. Even if they may already have had some ideas about the taxonomies before the intervention, they might have failed to create cognitively demanding tasks. PMTs in Group 1 indicated in their reflections they became aware of the level of cognitive demands of the HSE exam questions, they became more comfortable with successfully classifying HSE exam algebra questions according to the specified taxonomies, and they noticed how the questions should be written at the specified level during the intervention. That is, using two different categorizations might have helped them examine and interpret the level of questions from different perspectives and make more accurate judgments about their cognitive levels. In the teacher noticing context, various studies highlight the importance of using mathematical frameworks (e.g., Ivars, Fernandez, & Llinares, 2020; Walkoe, 2015). For instance, Walkoe (2015) explored teachers' noticing of student algebraic thinking in the context of video club discussions, and she used an algebraic thinking framework (ATF) to support teachers' noticing student algebraic thinking as part of a video club intervention. The results of the study showed that using the ATF was an effective strategy for improving teachers' noticing of student algebraic thinking. Similarly, in their study, Ivars et al. (2020) examined the role of a learning trajectory related to the part-whole meaning of the fraction concept to support pre-service primary teachers' noticing of students' mathematical understanding. The researcher indicated that the learning trajectory acted as a scaffold in pre-service teachers' noticing and provided them with a specific language to characterize students' understanding. With a similar interpretation, in this study, PMTs in Group 1 might have gained an in-depth understanding of HSE exam questions' level of cognitive demand while discussing the level of cognitive demands and providing justifications for their categorizations during the intervention through these frameworks. Therefore, PMTs' task knowledge, which includes examining and developing mathematically rich tasks considering students' learning needs, can be improved in their pre-service teacher education program.

Furthermore, both tasks developed by PMTs in Group 1 included more than one learning area, such as "algebra" and "numbers" or "algebra" and "geometry and measurement." To illustrate, in their second task, PMTs preferred to evaluate students' knowledge of performing operations with algebraic expressions using geometric shapes. Similarly, in many of the HSE exam questions, if the aim is to assess students' algebra knowledge, students are expected to do operations with algebraic expressions using geometric shapes. The collaborative examination of the objectives, algebraic content, and context addressed by the questions and spending sufficient time to do so during the intervention could be the factors in PMTs' reflections on the mathematical content and context of the HSE exam questions. PMTs collaboratively evaluated the questions beyond their superficial features by thinking of the questions' possible solutions and students' ways of thinking. For instance, while PMTs determined which items were algebra questions, they recognized that most of the 2018-2020 HSE exam algebra questions aimed to assess students' knowledge in other learning areas, particularly the "geometry and measurement" learning area. PMTs also collaboratively discussed whether or not HSE exam questions had a context during the intervention, and they recognized that many of the questions were contextual; particularly, they included daily life components. This may be a reason why they tended to prepare contextual tasks. Furthermore, the whole-class discussion under the instructor's guidance at the end of the sessions may also have been helpful in PMTs in Group 1's developing contextual and cognitively demanding tasks.

On the other hand, although PMTs in Group 1 developed cognitively demanding tasks, the level of their cognitive demand was not higher than the *analyze* level. They did not develop tasks at *evaluate* or *create* levels according to Bloom's revised taxonomy. PMTs might have been influenced by the cognitive level of HSE exam questions they examined during the intervention, thus preparing their tasks at similar levels since *apply* and *analyze* were the levels that the PMTs encountered most frequently as they examined sample items. Moreover, the problem context of T2 was found to lack clarity in some aspects. This finding may also be related to the limited time that PMTs had to develop their tasks. This study suggests that more time and experience are needed for PMTs to be able to develop more structured tasks at higher levels.

The findings also showed that two algebra tasks developed by PMTs in Group 1 were consistent with their reflections on the characteristics of HSE exam questions. They used several characteristics of HSE exam questions in their tasks they developed that they observed and noticed during the intervention. On the other hand, the second task was relatively more consistent with HSE exam questions regarding task structure and context. The first task, which was about basal metabolic rate (BMR), was very long structured, and it included more than one table and open-ended sub-question. The context of this task was also more realistic, and understanding the context was essential to solving this task. The reason why PMTs in Group 1 developed a long-structured task was most likely related to the researchers' request from PMTs to develop open-ended algebra tasks that were cognitively demanding.

Another finding was that PMTs in Group 1 noticed and developed tasks focusing on both core aspects of algebra according to Kaput (2008). This is important given that both core aspects are significant for developing students' algebraic thinking (Kaput, 2008). It is noteworthy that PMTs developed a task that not only used algebra as manipulation but also as formulating an equation. In other words, in T2, they addressed modeling the problem situation symbolically. Kieran (2004) suggested that algebraic thinking should involve not only a focus on solving a problem but representing it as well.

This research was limited to the algebra learning area. In future studies, PMTs can examine HSE exam questions in other learning areas, such as geometry and measurement, data analysis, and probability, and develop tasks in these learning areas. This research was also limited to PMTs' investigation of the HSE exam questions used in three years (2018-2020) since only three HSE exams were implemented when this research was conducted. Future studies should be conducted over a longer period, where PMTs can work on and examine more exam questions. The research findings were limited to one group of PMTs who participated in the intervention. This study only aimed to raise individual reflections of four PMTs working in a group on the characteristics of HSE exams' mathematics questions and to examine the characteristics of tasks developed by PMTs within the scope of HSE exam questions. Future studies may investigate the reflections of all PMTs who participated in the intervention, on different skill-based questions instead of only HSE exam questions and their competence to develop skill-based questions. Some studies suggested that the cyclic nature of the task development process supported by feedback increases the quality of tasks. Especially, feedback provided by different stakeholders such as researchers, peers, or students has a significant role in the improvement of task quality (e.g., Kaplan-Can, 2023). However, in this study, during or after PMTs presented the tasks they developed in class, the researchers and/or peers did not give detailed feedback for PMTs' tasks, and the researchers did not ask PMTs to create second versions of the tasks they developed. For that reason, they could not revise their tasks to increase their level of cognitive demands. Hence, in future studies, the developed and revised tasks of PMTs as a result of researchers' or peers' feedback can be examined in terms of task characteristics. In addition, PMTs can also be asked to apply their tasks to the middle school students since engaging in a task development process considering students' perspectives increases the quality of the tasks developed by the pre-service teachers (e.g., Chapman, 2004; Crespo, 2003; Kaplan-Can, 2023; Norton & Kastberg, 2012). Furthermore, in the analysis of the tasks they developed, PMTs' individual roles in group dynamics and their individual levels were not examined; instead, the tasks they developed were evaluated as a group. For this reason, future studies may focus on PMTs' individual roles in the group and their individual levels in terms of examined variables.

Consequently, this study suggests that mathematics teacher educators should provide opportunities for PMTs to examine cognitively demanding tasks and develop such tasks in their courses. The more PMTs review and develop cognitively demanding tasks before starting their profession, the more they might tend to use such tasks in their mathematics teaching practices. The PMTs' experiences presented in this study during their intervention may also provide mathematics teachers with an idea of the high-level task development process for teaching or assessment purposes.

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Appendix 1. Course Timetable

Week	Topic
1	Introduction to the course
	Proportional Reasoning
2	Proportional Reasoning
3	Algebraic Thinking: Generalization, Patterns, and Functions
4	Algebraic Thinking: Generalization, Patterns, and Functions
5	Algebraic Thinking: Generalization, Patterns, and Functions
6	Intervention
7	Intervention
8	Intervention
	Developing Concepts of Data Analysis
9	Developing Concepts of Data Analysis
10	Developing Concepts of Data Analysis
11	Exploring Concepts of Probability
12	Exploring Concepts of Probability
13	Exploring Concepts of Probability
14	Wrap-up

Appendix 2. The example of the table that asked PMTs to fill out by classifying 2018 HSE exam algebra questions

Dear Students,

Please examine the HSE-2018 mathematics questions given to you and determine the questions you think are related to "Algebra." Write down the question numbers you have determined in the table below and explain your opinion about why it is an algebra question. At the same time, analyze these questions according to the other headings given in the table and write your explanations in the relevant places. If you need, you can expand the table and add rows.

HSE EXAM 2018 QUESTIONS					
Question Number	**Why is this task an algebra question?**	**Is there a Context?**	**Learning Objectives (5-8) according to the middle school mathematics curriculum (MoNE, 2018b) (Note: One question may correspond to more than one objective.)**	**The level of cognitive demand according to Smith and Stein (1998)**	**The level of cognitive demand according to Bloom's Revised Taxonomy**

*The same table was used in the examination of HSE Exam 2019 and 2020 questions.
**This table was not used as a data collection tool, and PMTs filled the tables during the intervention to classify the HSE exam questions related to algebra.