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Original Article

Identification of Number Sense Strategies Used by Eighth-Grade Students in Fractions Questions in Figure, Operation, and Scenario Forms

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Abstract

The aim of this study is to determine the number sense strategies used by eighth-grade students with different academic achievement levels in fractions questions in figure, operation, and scenario forms. The study used the case study design, which is a qualitative research method. The study group consists of three eighth-grade students attending a public school in Istanbul. The students participating in the study were determined by purposive sampling. As a data collection tool, the study employed the Number Sense Test consisting of 18 questions in figure, operation, and scenario forms prepared by the researchers. The data were collected through semi-structured interviews synchronously through video conferencing. The results indicated that the students used number sense-based solution strategies in the questions in figure form the most, which were followed by the scenario questions and operation questions, respectively. In addition, the student with low academic achievement used more number sense-based solution strategies in the questions in figure and scenario forms than the student with medium academic achievement. The students applied more number sense-based solution strategies in figure and operation questions in the component of understanding of the meaning and size of numbers; in figure questions in the component of understanding and use of equivalent expressions; and in scenario questions in the component of understanding the meaning and effect of operations. Moreover, although the component of flexible computing and deciding the reasonableness of the result was applied in the questions in scenario form the most, the most mistakes were made in a scenario question as well. Finally, the use of number sense acted as a bridge between students' intuitive thinking and conceptual knowledge in learning fractions. Based on the obtained results, suggestions are made for future studies.

Keywords

Fraction
Number sense
Figure form
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Scenario form

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Introduction

How many days have passed since the day you were born? How many minutes does it take to walk a 5-kilometer road on average? When we encounter questions like “When you take your own height as a reference, what is the height of the building where you live in terms of your height?”, we are expected to be able to answer without resorting to the paper-pencil algorithm. In order to solve such problems that we encounter in our daily life, we need skills such as estimating, knowing the meaning and magnitude of numbers, flexible thinking, and using comparison (reference) points in problem situations, instead of performing algorithm-based operations using paper and pencil. These skills that we are expected to have are described with the concept of number sense in the literature (Kayhan Altay, 2010; National Council of Teachers of Mathematics [NCTM], 2000; Reys & Yang, 1998).

Number sense is a construct that includes skills such as understanding number systems and relations between numbers, making logical predictions, using various reasoning strategies, and making flexible calculations (McIntosh, Reys, & Reys, 1992; Shumway, 2011; Yang & Sianturi, 2019a, 2019b). Gülbağcı Dede and Şengül (2016) emphasize that individuals with number sense can rationally interpret and make sense of situations involving numbers and operations and can determine the most appropriate solutions for the problems they encounter and use them flexibly. For example, even if a student with number sense does not know the algorithm for division by fractions, s/he can say that the answer is 7 when finding the result of the operation $3\frac{1}{2}:\frac{1}{2}$, thinking that the fraction $3\frac{1}{2}$ has 7 halves (Van de Walle, Karp, & Williams, 2014), or when asked about 3.91 x 0.95, s/he can estimate that the result will be smaller than 3.91 because 0.95 is smaller than 1 (Graeber & Tirosh, 1990; Greer, 1987; McIntosh et al., 1992). In other words, s/he may feel that multiplication does not always make numbers bigger, and division does not always make numbers smaller. In this context, it can be said that number sense is very important for students to construct the meaning and effects of operations without memorizing algorithms. Many studies in the literature highlight that students who memorize fractions based on algorithms have a very weak understanding of the concept of fractions (Ball, 1990; Bush & Karp, 2013; Gabriel, Coche, Szucs, Carette, Rey, & Content, 2013; Hecht & Vagi, 2012; Yang & Huang, 2004) and a high rate of making mistakes in the fractions questions (Clarke & Roche, 2009; Ni & Zhou, 2005; Riddle & Rodzwell, 2000; Siegler & Pyke, 2013; Tirosh, 2000; Van Hoof, Van de Walle, Verschaffel, & Van Dooren, 2014).

Riddle and Rodzwell (2000) report that when they asked the fourth-grade students who had not yet learned the subject of operations with fractions the result of the operation $2\frac{1}{2}+\frac{3}{4}$, 40% of the students considered the fraction $\frac{1}{2}$ as 2 times $\frac{1}{4}$ and made them integer, thus answering the question correctly. On the other hand, 22% of the fifth-grade students who took some lessons on fractions used the strategy of finding a common denominator, but none of them could solve the question successfully. Clarke and Roche (2009) found that when the sixth-grade students compared the fractions $\frac{3}{4}$ and $\frac{7}{9}$, only 10% gave correct answers and 40% of the students who gave correct answers made comparisons by considering the closeness of both fractions to whole. According to the National Assessment of Educational Progress (NAEP) report published in 1978, when asked the approximate result of the operation $\frac{12}{13}+\frac{7}{8}$, only 24% of the students gave the answer 2, and the most common answer was 19 (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). In the report published in 2014, only 27% of the students answered the same question correctly (Lortie-Forgues, Tion, & Siegler, 2015). These results clearly show that memorized rules and algorithms do not always lead students to correct results, and students have difficulties in using number sense. However, number sense is very important in mathematics education and especially in learning fractions, and it is referred to as one of the basic concepts in mathematics (Dekker & Dolk, 2011; Feigenson, Libertus, & Halberda, 2013; Mohamed & Johnny, 2010; NCTM, 2000; Östergren & Träff, 2013). The NAEP (2019) report states that number sense is one of the most important expectations in mathematics.

Despite the consensus among researchers regarding the place and importance of number sense in mathematics education, there is no common classification for number sense. In their literature review on number sense, Şengül and Gülbağcı Dede (2013) tried to reveal all the classifications of number sense. However, at the end of the study, they concluded that the boundaries of the concept of number sense could not be drawn clearly, and a common terminology could not be established for the components. For example, Greeno (1991) stated that number sense has three components: flexible numerical computation, computational estimation, and quantitative judgment and inference. However, the component of flexible numerical computation, defined by Greeno (1991) as "recognition of equivalences in order to regroup numbers in mental multiplication", was called by McIntosh et al. (1992) as "multiple representations for numbers" and by Yang (1995) as "decomposing and recomposing numbers". Similarly, the ability to understand the relative size of numbers was named by McIntosh et al. (1992) as "sense of relative and absolute magnitude of numbers" while Reys et al. (1999) named this component as "understanding the magnitude and meaning of numbers" and associated it with questions for comparing fraction sizes. Markovits and Sowder (1994) stated that understanding number magnitude requires skills such as comparing and ordering numbers. Thus, although there is no consensus on the components of number sense, there seems to be a consensus on understanding operations and relationships between them in depth, performing flexible operations with numbers, and applying number knowledge to numerical situations. From this perspective, Reys et al. (1999) developed a framework consisting of six components that are agreed upon in the literature. The component of "understanding and use of equivalent representations of numbers", which is the sixth one in the related study, is treated within the component of "understanding and use of equivalent expressions" in the present study, thus not being considered as a separate component. The five components addressed in this study are explained in relation to the subject of fractions below:

- Understanding of the meaning and size of numbers: It is related to the ability to recognize the relative size of numbers (McIntosh et al., 1992; Şengül, 2013). For example, finding the result $\frac{14}{15} > \frac{10}{11}$ as $\frac{1}{11} > \frac{1}{15}$ considering the distance of the fractions $\frac{10}{11}$ and $\frac{14}{15}$ to whole is related to this component (Der-Ching & Hung-Jin, 2019).
- Understanding and use of equivalent expressions: It is the ability to know and use equivalent numbers when necessary (Şengül, 2013). For instance, being aware of that the fraction $\frac{1}{4}$ can be represented by forms such as $\frac{2}{8}$, 25%, and 0.25% is related to this component (Lin, Yang, & Li, 2016).
- Flexible computing and deciding the reasonableness of the result: It refers to solving problems without written calculations, and applying a mental estimation strategy (McIntosh et al., 1992). For example, realizing that the sum of three fractions that are smaller than $\frac{1}{3}$ is smaller than 1 is within the scope of this component (Gülbağcı Dede & Şengül, 2016).
- Understanding the meaning and effect of operations: It is the ability to understand how the result will change when operations or numbers are changed in calculations (McIntosh et al., 1992; Yang, 2005). For instance, in the operation $\frac{14}{25} \times \frac{7}{17}$, deciding that the result is smaller than $\frac{1}{2}$ by considering that the fraction $\frac{14}{25}$ is smaller than 1 and the fraction $\frac{7}{17}$ is smaller than $\frac{1}{2}$ is related to this component (Lin et al., 2016).
- Measurement benchmarks: It is the ability to determine and use reference points (McIntosh et al., 1992). For example, being able to estimate the length of a football field by taking one's own height as a reference is related to this component (Şengül, 2013).

Purpose and Significance of the Study

The literature contains many studies aimed at identification of the number sense strategies of students and pre-service teachers (Aksakal, 2020; Aktaş & Özdemir, 2017; Can & Yetkin Özdemir, 2020; Gülbağcı Dede & Şengül, 2016; Şengül & Gülbağcı Dede, 2014; Şengül, 2013; Whitacre & Nickerson, 2016; Yang & Hsu, 2009; Yang & Huang, 2004; Yang, 2005, 2006, 2007; Yang, Reys, & Reys, 2009; Yenilmez & Yıldız, 2018), development of number sense in students (Yang, 2006), and explanation of the relationship between number sense and different variables (Can, 2019; Harç, 2010; Mohamed & Johnny, 2010; Yang, 2005; Yang, Li, & Lin, 2008). One of the common results of the studies examining the number sense strategies of students and pre-service teachers is that number sense performance is low and the tendency to rule-based solutions is high, while another is that the use of number sense strategies varies depending on number sense components (Aksakal, 2020; Aktaş & Özdemir, 2017; Can & Yetkin Özdemir, 2020; Harç, 2010; Gülbağcı Dede & Şengül, 2016; Şengül & Gülbağcı Dede, 2014; Şengül, 2013; Takır, 2016; Whitacre & Nickerson, 2016; Yang & Hsu, 2009; Yang & Huang, 2004; Yang, 2005, 2007; Yang et al., 2009; Yenilmez & Yıldız, 2018).

Depending on sample, subject area, or questions, some previous studies determined that students used number sense-based solution strategies more in the component of understanding the meaning and effect of operations (Yenilmez & Yıldız, 2018), while some others detected more use of number sense-based solution strategies in the component of measurement benchmarks (Harç, 2010). As to the data collection tools, Yenilmez and Yıldız (2018) used questions in operation form for the component of understanding the meaning and effect of operations, and Harç (2010) employed questions in figure form for the component of measurement benchmarks. Can and Yetkin Özdemir (2020) observed, on the other hand, that the use of number sense by the students differed significantly only in the context question in figure form. Similarly, the international literature contains studies addressing the use of number sense within the context of question forms. Greenes, Schulman, and Spungin (1993) suggested fill-in-the-blanks/scenario completion activities to improve students' number sense and emphasized that students' ability to verify whether their number choices are reasonable and to understand mathematical relationships would improve through such activities. Yang (2006), on the other hand, aimed to improve the number sense of fourth-grade students by using one of the scenario completion activities suggested by Greenes et al. (1993) and concluded that number sense skills can improve depending on the context used. Yang and Hsu (2009) highlighted that questions in operation form such as "Without calculating and in the absence of the exact answer, best estimate for $\frac{15}{16} + \frac{11}{12}$" contributed to the improvement of students' number sense. These results obtained from the related studies show that students' use of number sense differs depending on number sense components, and the structure of the question form used affects students' number sense performance (Can & Yetkin Özdemir, 2020; Gülbağcı Dede & Şengül, 2016; Greenes et al., 1993; İymen & Duatepe-Paksu, 2015; Yang & Hsu, 2009; Yang, 2006).

In addition, many studies examining the relationship between number sense and various variables observed that number sense differs significantly based on mathematics achievement (Harç, 2010; Mohamed & Johnny, 2010; Yang, 2005; Yang et al., 2008; Yenilmez & Yıldız, 2018). Harç (2010) and Yang (2005), asking sixth-grade students questions in figure and operation forms, found that students with high mathematics achievement used more number sense strategies than other students. Similarly, Yang et al. (2008) emphasized that there is a significant relationship between fifth-grade students' number sense and their mathematics achievement. Moreover, they concluded that the students showed the best performance in the question in operation form in the component of understanding the meaning and size of numbers. However, Can (2019), investigating the use of number sense by fourth-grade students in context and non-context problems, revealed that the students with low academic achievement performed similarly to those with medium and high academic achievement.

When the studies in the literature mentioned above are considered together, the different results obtained may have derived from the structures of the question forms used. On the other hand, no study has been found in the literature that comparatively examines the use of number sense by students with different achievement levels in terms of question forms. In this respect, examining the use of number sense by students with different achievement levels in different question forms is significant both to fill the gap in the literature and to contribute to the improvement of students' number sense. Also, exploring the use of number sense in questions in different forms is valuable in terms of guiding teachers and curriculum developers. Moreover, considering the place of fractions in mathematics education and the importance of using number sense in this subject, the study is anticipated to contribute to the teaching of fractions through number sense.

Given the reasons above, the present study aims to examine the number sense strategies used by eighth-grade students with different achievement levels in fractions questions in figure, operation, and scenario forms. Hence, the study seeks answers to the following questions:

1. How do eighth-grade students with different academic achievement levels use number sense strategies in different forms of fractions questions?
2. What are the differences between the number sense strategies used by eighth-grade students with different academic achievement levels in different forms of fractions questions?

Method

This study employed the case study model, which is a qualitative research method. Case study is an approach in which a limited system (a situation) is described and examined in detail and in depth through multiple sources of information (Creswell, 2018). The present study is a case study as it aims to examine the number sense skills of eighth-grade students with different academic achievement levels in different forms of fractions questions in detail and in depth.

Study Group

The study group consists of three eighth-grade students in a middle school in Istanbul in the 2020-2021 academic year, determined through purposive sampling. The reason for choosing the purposive sampling method is that this method allows to identify a specific person or a group that can best respond to the problem by selecting information-rich situations in line with the purpose of the study (Creswell, 2018). For this reason, the study was conducted with three eighth-grade students with different academic achievement levels chosen based on teacher views and in-class observations and thought to be able to give the most appropriate responses to the research problem. This is because all the achievements in the primary education mathematics curriculum targeted for the study were covered by the students at this grade level and were associated with different subject areas. For research ethics, the study group was created on a voluntary basis, and consent forms were obtained from the students and parents. In addition, the real names of the participants were not used.

The students in the study group had been studying in the same class since the fifth grade, and their achievement levels were different from each other. The achievement levels of the students were classified based on their grade point averages in the fifth, sixth, and seventh grades, as indicated in Table 1.

Table 1. Student Code Names by Achievement Level

Student Code Names	Achievement Levels	Grade Point Averages
Ali	High Achievement (HA)	98.86
İnci	Medium Achievement (MA)	80.55
Su	Low Achievement (LA)	52.77

Data Collection Tools

In qualitative studies, data should be collected by using various data collection tools by their nature and to ensure reliability (Merriam, 2015). The data of this study were collected using the Number Sense Test (see Appendix-1) developed by the researchers and the semi-structured interviews. Since the semi-structured interviews were conducted through video conferencing, video and audio recordings were also taken.

The fractions questions in the Number Sense Test were created based on the components of (i) understanding of the meaning and size of numbers, (ii) understanding and use of equivalent expressions, (iii) flexible computing and deciding the reasonableness of the result, (iv) understanding the meaning and effect of operations, and (v) measurement benchmarks. Table 2 summarizes the achievements and components associated with the questions given in Appendix-1 in detail.

Table 2. The Achievements for the Questions Prepared for the Number Sense Components

Number Sense Component	Achievement	Question Numbers *
Understanding of the meaning and size of numbers	Compares, orders, and displays fractions on the number line	F-1, O-1, S-1
Understanding and use of equivalent expressions	Compares, orders, and displays fractions on the number line	F-2, O-2, S-2
Flexible computing and deciding the reasonableness of the result	Performs addition and subtraction with fractions. **	F-3, O-3, S-3 F-4, O-4, S-4
Understanding the meaning and effect of operations	Performs multiplication with fractions.	F-5, O-5, S-5
Measurement benchmarks	Performs division with fractions.	F-6, O-6, S-6

* In the abbreviations in the question numbers, F stands for figure form, O for operation form, and S for scenario form.

** In this component, addition and subtraction were handled separately, and different forms of two separate questions were asked.

In line with the achievements associated with each number sense component described in Table 2, the questions in which the same numbers were used were prepared in different formats, i.e., in figure, operation, and scenario forms. Opinions of two mathematics educators and a faculty member expert in the relevant field were taken regarding the questions in the data collection tool. First, two mathematics educators were asked whether the questions in the Number Sense Test were consistent with the targeted achievements, and then the field expert was asked whether the questions represented different types of number sense components. The experts also checked the wording of the questions, their degree of difficulty, and the extent to which they measured what they were intended to measure. Based on the expert opinions, the questions prepared for the components of (iii) flexible computing and deciding the reasonableness of the result and (v) measurement benchmarks were restructured on the grounds that they did not represent different forms of the same question. In addition, in order to minimize random success in the questions in scenario form, the number of choices given to the students was one more than the number of blanks left. The questions in figure and operation forms were prepared with multiple choice to increase the students' possibility of alternative thinking. After the changes, expert opinion was taken again, and the Number Sense Test was finalized (see Appendix-1).

Question in figure form: It is the type of question in which fractions are represented by figures. The questions in figure form used in the study were prepared by the researchers in line with the studies and suggestions in the literature (Van de Walle et al., 2014; Yang & Hsu, 2009).

Question in operation form: It is the mathematical question form of comparison, addition-subtraction, multiplication-division operations for fractions. Among the questions in operation form used in the study, O-4 (see. Appendix-1) was taken directly from the literature (NAEP, 1978, as cited in Carpenter et al.,1980; NAEP, 2014, as cited in Lortie et al., 2015), and the other questions in operation form were prepared based on the studies in the literature (Lin et al., 2016; Markovits & Sowder, 1994; Yang, 2007; Yang & Hsu, 2009; Zanzali & Ghazali, 1999).

Question in scenario form: It is the type of question in which a scenario is given to the students on the subject of fractions, and the blanks in the text are filled to ensure the integrity of the meaning in the scenario. In this study, scenarios were prepared in different contexts for each number sense component. Scenarios in which the stories "Lucky Luke", "Around the World in Eighty Days", and "Alice's Adventures in Wonderland" were associated with the learning domain of fractions were created for the components of understanding of the meaning and size of numbers, flexible computing and deciding the reasonableness of the result, and understanding the meaning and effect of the operations, respectively. Since the contexts of the said stories were consistent with the targeted achievements (Van de Walle et al., 2014), the scenarios were prepared within the framework of these stories. Scenarios for the components of measurement benchmarks and understanding and use of equivalent expressions were designed by the researchers in a real-life context.

Procedure

Three different experiment forms were prepared based on the Number Sense Test developed in the study. Table 3 presents the distribution of the questions in each experiment by question form. In each experiment, six questions were asked in each form: two in operation form, two in figure form, and two in scenario form, totaling in 18 questions.

Table 3. Distribution of Question Forms in Each Experiment

Question Number	Experiment-1	Experiment-2	Experiment-3
1	F-1	F-3	F-5
2	F-2	F-4	F-6
3	O-3	O-5	O-1
4	O-4	O-6	O-2
5	S-5	S-1	S-3
6	S-6	S-2	S-4

The data of the study were collected in three sessions. Each session was held at convenient times for the students, and Experiment-1 was conducted in the first session, Experiment-2 in the second session, and Experiment-3 in the third session. All three experiments were parallel to each other. Since the numbers used in the experiments were the same but the ways they were presented were different, the sessions were held at one-week intervals. In addition, one student was interviewed in each session. The students did not interact with each other. Table 4 shows which experiment was conducted in each session, as well as the session dates and durations of the interviews with the participants.

Table 4. Durations and Dates of the Interviews with the Participants

Session	Date	Ali	İnci	Su
Experiment-1	20.01.2021	45′ 11′′	50′ 57′′	36′ 35′′
Experiment-2	27.01.2021	40′ 54′′	38′ 37′′	35′ 25′′
Experiment-3	03.02.2021	39′ 27′′	53′ 40′′	43′ 57′′

Since the study was carried out in the Covid-19 distance education period, the data of the study were collected synchronously through video conferencing. The participants were informed that the sessions would be recorded by camera throughout the data collection process, and none of the participants felt uneasy. The data collection tool was shared with the students in the computer environment. The students were asked to solve the questions by thinking aloud, and it was stated that they could solve the questions as they wished on the computer screen. The students solved the questions on the computer screen by using the drawing tools in the video conferencing application. However, in some cases, especially when they resorted to rule-based solutions, they answered the questions using paper and pencil. Immediately after the student solved a question, an interview was held for that question. During the interview process, all students were asked to explain in detail how they solved the question. In addition, in order to determine whether the students using a rule-based solution method noticed the number sense-based solution, they were asked questions such as "Can you explain how you solved this problem?", "Can you solve the problem in a different way?". There was no time limitation during the experiments, and after the students were asked if there was anything they wanted to add about the question, they moved on to the new question.

Validity and Reliability

In qualitative research, taking valid measures to reach correct information refers to "validity", and defining the research process in a way that allows another researcher to evaluate it refers to "reliability" (Yıldırım & Şimşek, 2016). The validity and reliability of this study is explained based on the criteria determined by Guba and Lincoln (1982).

Credibility: As one of the researchers was also a practicing teacher, she had interacted with the students in the study group for a period of three years. She collected the data in three separate sessions in two weeks. As Houser (2015) states, being in the same environment all the time allows establishing a relationship based on mutual trust and receiving correct and complete answers. In this way, the participants gave sincere answers. In addition, at every stage of the research, expert opinion was taken from a faculty member teaching math and having national and international studies on number sense. The expert critically evaluated the whole process from the design of the study to the collected data, their analysis, and the writing of the results, and gave feedbacks.

Transferability: To ensure transferability, how the study group was selected, the characteristics of the participants, and the study environment are explained in detail. In addition, the results of the study are presented through direct quotations from the student solutions. The number sense strategies used by the students while answering the questions were examined in depth through the students' audio and written solutions and semi-structured interviews. The student solutions and audio recordings were recorded so that they could be submitted to researchers' review if requested.

Reliability: In case studies, reliability is ensured by clearly defining the research process, supporting it with documents, and seeking expert opinions (Öztuna Kaplan, 2013). In this study, the research process was clearly defined, supported by documents, and expert opinions were sought. Data triangulation was applied in data collection. Interviews with the students and their written responses were used as data collection tools. In addition, expert opinions were obtained from a field education expert at every stage of the study process and from three different mathematics educators during the development of the data collection tool. Thus, precautions were taken to avoid subjective judgments.

Data Analysis

In the data collection process, various data collection tools were used, including student notes and video and audio recordings of the semi-structured interviews. In the data analysis process, the audio recordings obtained during the semi-structured interviews were transcribed first. The transcribed data set was analyzed in two stages through categorical analysis, which is a content analysis technique. Categorical analysis refers to first dividing a particular message into units, and then grouping these units into categories based on certain criteria (Bilgin, 2006). In this study, student answers were categorized as "correct", "wrong", and "blank" in the first stage by a researcher continuing PhD study in mathematics education and a faculty member studying on number sense. In the second stage, the answers in the "correct" and "wrong" categories were examined in four categories based on the criteria in the study of Der-Ching and Hung-Jin (2019): number sense-based (NSB), partially number sense-based (PNSB), rule-based (RB), and erroneous explanation (EE). Table 5 presents, with examples, how and based on what criteria the student answers were evaluated.

Table 5. Evaluation of the Answers in the "Correct" and "Wrong" Categories

Solution Strategy	Evaluation Criteria	Example
Number Sense-Based Solution (NSB)	Uses numbers and operations flexibly without using algorithms.	(F-2) The answer "*The shaded area given in both figures is a quarter of the whole, so both fractions are equal*" was considered as NSB, as it shows that equivalent expressions could be understood and used without depending on rules and algorithms.
Partially Number Sense-Based Solution (PNSB)	Also uses some rules and formulas despite using the concept of number sense to solve the problem.	(F-1) The answer "*I'll depart from the choices and equalize the denominators of all the fractions and order them in ascending order. The choices A and D are appropriate, but it asks for the most appropriate. Then I'll go for considering closeness to half*" was considered as PNSB, since it shows that the rule-based solution strategy was used first, but as that solution was not sufficient to make a decision, number sense-based solution was applied in the rest.
Rule-Based Solution (RB)	Performs standard calculations. Reaches the result by following certain algorithms.	(S-1) The answer "*I equated the denominators of the numbers and ordered them according to the information given in the text*" was considered as RB, since only standard calculations were made.
Erroneous Explanation (EE)	Answers the question correctly, but makes meaningless explanations. Answers the question incorrectly and makes meaningless explanations.	(F-5) The answer "All choices have 100, but C has 50. I choose C" was considered as EE, since it had no mathematical basis.

After the student answers were categorized as "correct" and "wrong" and their uses of number sense were determined, the uses of number sense in questions in different forms were examined based on each number sense component. In this context, the agreement in the evaluations of the researchers was calculated based on Miles and Huberman's (1994) percentage of agreement = [agreement / (agreement + disagreement)] x 100. As a result of this calculation, the percentage of agreement was found to be 90%. A percentage of over 70% is regarded reliable for research (Miles & Huberman, 1994). Although a reliable percentage was obtained, the two researchers and the expert performing the data analysis came together again and discussed the points of disagreement until an agreement was reached. The researchers had a disagreement especially in the PNSB solution category, and the student's considering the closeness of fractions to half or whole after equating the denominators was coded as NSB by one of the coders and as PNSB by the other. However, it was decided to code the solution as PNSB when the student resorted to the algorithm even once while solving the problem. In cases where both researchers were indecisive, opinions were received from two mathematics educators who were informed about number sense, only in the context of the relevant questions. After the data were coded as specified, they were evaluated quantitatively on the basis of percentage and frequency. In the results section, direct quotations are made regarding the strategies used by the participants in the solution of the questions, thereby examining the students' use of number sense in depth.

Results

The results obtained in the study are presented in the order of sub-problems below.

Results for the First Sub-Problem

In line with the first sub-problem of the study, it was aimed to examine the number sense strategies used by the students with different academic achievement levels in different forms of questions. Table 6 presents the distribution of the strategies used in solving the questions in figure, operation, and scenario forms based on the NSB, PNSB, RB, and EE categories. In this context, 54 answers given by three students with different academic achievement levels to 18 questions in different forms are analyzed and presented below.

Table 6. Distribution of the Solution Strategies Used for the Question Forms by Academic Achievement

Question Form	**Achievement Level**	**Mathematical Accuracy (Frequency/Percentage [%])**							
		Correct			**Wrong**				**Blank**
		NSB	PNSB	RB	NSB	PNSB	RB	EE	
Figure Form	Ali (HA)	**4**(22.2)	2(11.1)	-	-	-	-	-	-
	İnci (MA)	**1**(5.5)	2(11.1)	**1** (5.5)	-	1(5.5)	1(5.5)	-	
	Su (LA)	**2** (11.1)	-	-	1(5.5)	-	1(5.5)	**2**(11.1)	-
	Total	**7(38.8)***	**4(22.2)**	**1(5.5)***	**1(5.5)**	**1(5.5)**	**2(11.1)**	**2**(11.1)	
Operation Form	Ali (HA)	2(11.1)	3(16.6)	1(5.5)	-	-	-	-	-
	İnci (MA)	1(5.5)	1(5.5)	4(22.2)	-	-	-	-	-
	Su (LA)	-	-	-	1(5.5)	1(5.5)	3(16.6)	-	1(%17)
	Total	**3(16.6***	**4(22.2)**	**5(27.7) ***	**1(5.5)**	**1(5.5)**	**3(16.6)**	**2**(11.1)	
Scenario Form	Ali (HA)	**3 (16.6)**	2(11.1)	1(5.5)	-	-	-	-	-
	İnci (MA)	-	-	3(16.6)	-	-	3(16.6)	-	-
	Su (LA)	**1** (5.5)	-	-	-	-	1(5.5)	3(16.6)	1(5.5)
	Total	**4(22.2)***	**2(11.1)**	**4(22.2)**	**-**	**-**	**4(22.2)**	**3(16.6)**	**1(5.5)**

When the correct answers given by the students to the questions in different forms in Table 6 are compared in terms of NSB solution strategy, the order of percentage is as follows: figure form (38.8%), scenario form (22.2%), and operation form (16.6%). The students applied RB solution strategy mostly in the questions in operation form (27.7%). In addition, the students gave the most wrong answers to the questions they solved with RB solution strategy. When the strategies used by the students in the questions in different forms are examined in terms of academic achievement, it is clearly seen in Table 6 that the student who gave correct answers to all questions, whether in figure, operation, or scenario form, and who used NSB solution strategy was Ali (HA). While answering the questions in figure and scenario forms, Su (LA) applied NSB solution strategy more than İnci (MA). On the other hand, it is noteworthy that Su (LA) answered the questions in scenario (two questions) and figure (one question) forms correctly by using NSB solution strategy, while answering all questions in operation form incorrectly.

Results for the Second Sub-Problem

To examine the solution strategies used by the students in figure, operation, and scenario forms in depth, the sample answers and the student views obtained from the semi-structured interviews are explained comparatively based on number sense components.

a) Understanding of the meaning and size of numbers

Table 7 below presents the solution strategies used by the students while answering the questions prepared in different forms for the achievement of "compares, orders, and displays fractions on the number line" in the component of "understanding of the meaning and size of numbers".

Table 7. Students' Solution Strategies in the Component of Understanding of the Meaning and Size of Numbers

	Figure Form		Operation Form		Scenario Form	
	Correct	**Wrong**	**Correct**	**Wrong**	**Correct**	**Wrong**
Ali (HA)	PNSB		PNSB		PNSB	
İnci (MA)	PNSB		PNSB		RB	
Su (LA)		RB		PNSB		RB

When the answers of the students in Table 7 are evaluated in terms of the solution strategies they used, it can be said that Ali (HA) and İnci (MA) predominantly used PNSB solution strategy and Su (LA) used RB solution strategy, regardless of question form. In addition, while Ali and İnci answered all questions correctly, Su gave incorrect answers. To examine the difference in the students' answers in depth, the answers given by each student to the questions in figure, operation, and scenario forms are presented comparatively in Table 8.

Table 8. Student Answers in the Component of Understanding of the Meaning and Size of Numbers

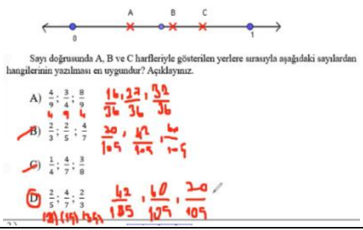
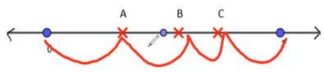
Figure Form		
Ali (PNSB)	**İnci (PNSB)**	**Su (RB)**
A: …I'll equalize the denominators of all and list them in ascending order. The choices A and D are appropriate, but it asks for the most appropriate. Then (choice D) I'll go for considering closeness to half. 42/105 is smaller than half, 60/105 is slightly larger than half, which is as ***it should be, 70/105 is larger than half****… So D…* *Researcher: How did you decide between A and D?* *A: The difference between B and C needs to be less. But, if we think of it as 36/36 in A, there should be a gap of 4 units between C and 1, while the gap between B and C should be less.*	*First, I equated the denominators. They are already ordered from smallest to largest. That's why C and B can't be the answers. Both A and D are appropriate. But well (for choice A) B has to be close to half but larger than half. So, it should be close to 18/36, but 27/36 is close to whole. So, A can't be the answer. The answer is D.* 	*S: Will we count the intervals?* *R: What will you find when you count the intervals?* *S: So, sir, there are no numbers here… There are 3 intervals up to B, since there will be 4 in total, B must be ¾. …* *R: … how can we find A and C?* *S: I don't know… I'd say A.* 

Table 8. Continued

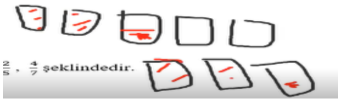
Operation Form		
Ali (PNSB)	**İnci (PNSB)**	**Su (PNSB)**
Choice A is wrong because 2/3 is larger than half and 2/5 is smaller than half. *Choice B is wrong, 4/7 is closer to whole.* *In choice C, the fraction 2/5 is closer to half than the fraction 2/3. Now we have 2.5/5 and 1.5/3, how much is there from both to half? I think I'll equate the denominators. Then come 6/15 and 10/15. While there's a 1.5 difference from 6/15 to half, there's a 2.5 difference in 10/15, which means that 6/15, that is 2/5 is closer to half. So, that would be correct.* *In choice D, 2/5 is still not half, but 4/7 is over half, so D is wrong. I found the answer as C.*	*Choice A is wrong because if we* ***follow the rule*** *that the smaller is the denominator, the larger is the number, it will be wrong.* *In choice B, I* ***expanded****. 4/7 gets closer to whole. In other words, 17.5/35 would be just the middle, but* ***30/35 is over half, while the other is not****.* *In C, we equate the denominator. 6/15 and 10/15 (half 7.5). The first one is not half, and the other one is over half. That's why it's correct.* *In D,* ***I equated the denominator. The larger the numerator, the larger is the number****.*	*In choice A, if we evaluate it based on the cake, a larger cake comes, so 2/3 is larger. A is wrong.* *In choice B, in both 2/5 and 4/7,* ***the difference is 3****, so shouldn't they be equal? So, this is also wrong.* *In choice C, the fraction 2/3 is closer to half than 2/5.* ***Let me show with the box.***  *In choice D, I've already said above that 2/3 is larger. If we eat cake 2/5 and 4/7,* ***when I divide it by 5, the slice will be larger.*** *So, 2/5 is larger. Answer is D.* *Researcher: Can you explain again how you decided on choices C and D?* *S: I've already drawn it in C. 2/5 is less than half, but 2/3 is over it. 2/3 is closer. In D, I considered it like a cake and thought which one would have the larger slice.*
Scenario Form		
Ali (PNSB)	**İnci (RB)**	**Su (RB)**
… Since Joe is the shortest, I have to find the smallest fraction. The smallest is 2/5. This is because other fractions are larger than half, but 2/5 is smaller than half.… 2/3 and 4/7, no way out. Both can be possible. Both are larger than half, so I equalized the denominators. There come out 14/21 and 12/21. So, we can say 2/3 > 4/7.	*I eliminated 3/2 straight away. Because if we say 3/2, Joe will be taller. Actually, I* ***equalized the denominators*** *of the remaining numbers, and then* ***I ordered them based on the information given in the text****.*	*S: I want to make 2/3, since Joe is shorter. This is because the difference is shorter.* *A: Why did you look at the difference between the numerator and the denominator?* *S: Because I'm comparing. If the difference is small, it should be smaller. 2/5 to the 2nd blank, because the difference between them is 3. 3/2 to the last blank, because 3 is larger than 2…*

Table 8 shows that Ali and İnci tended to compare the fractions by equalizing the denominators in the question in figure form. However, in cases where it was not sufficient to make a decision by equating the denominators, they compared the fractions by considering their closeness to half and whole. Considering the answers of all three students in scenario form, it is noteworthy that while Ali was comparing the fractions that were both larger than half, he determined their distances to half, but he could not interpret them. İnci, on the other hand, was aware that a compound fraction of a natural number would be larger than that number, but she used RB solution strategy to compare the remaining fractions.

Su, who had a low academic achievement, could not compare fractions in all three forms due to her imperfect conceptual knowledge about fractions. In the question in figure form, the student focused only on the intervals given on the number line, ignoring that fraction is a whole divided into equal parts or a cluster composed of discrete equal parts. Similarly, she compared the fractions by considering only the meaning of the denominator in the question in operation form and by thinking that the distance to whole is the difference between the numerator and the denominator in the question in scenario form. In other words, she treated the difference between the numerator and the denominator as a natural number and made a comparison accordingly.

b) Understanding and use of equivalent expressions

Table 9 below presents the solution strategies used by the students while answering the questions in different forms in the component of "understanding and use of equivalent expressions".

Table 9. Students' Solution Strategies in the Component of Understanding and Use of Equivalent Expressions

	Figure Form		Operation Form		Scenario Form	
	Correct	**Wrong**	**Correct**	**Wrong**	**Correct**	**Wrong**
Ali (HA)	NSB		NSB		NSB	
İnci (MA)	PNSB		RB			RB
Su (DB)	NSB			NSB		EE

Table 9 shows that Ali (HA) used NSB solution strategy in all question forms, while İnci (MA) used PNSB in figure form and RB solution strategy in operation and scenario forms. On the other hand, Su (LA) applied NSB solution strategy in questions in figure and operation forms, but only answered the question in figure form correctly. Table 9 indicates that the only question answered correctly by all students is in figure form. To examine these results in depth, the sample student answers and the dialogues between the researcher and the student are presented in Table 10 with direct quotations.

Table 10. Student Answers in the Component of Understanding and Use of Equivalent Expressions

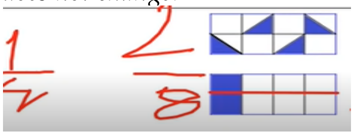
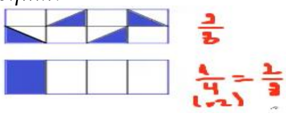
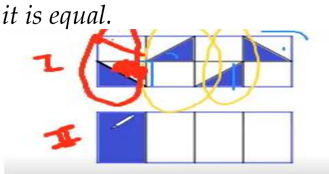
Figure Form		
Ali (NSB)	**İnci (PNSB)**	**Su (NSB)**
The shaded area given in both figures is a quarter of the whole. *We can also think of it this way: if we divide the second figure in half, we get the top figure again. Also, when we expand and simplify a fraction with the same number, the result does not change.* 	*İ: If we expand this fraction (1/4), the fraction becomes 2/8, so the two are equal.*  *A: Why do we get two equal fractions when we expand?* *İ: When I multiply both the numerator and the denominator by the same number, I get an equivalent fraction. There can be no reason for this…* *A: Can you solve the question in a different way?* *İ: I can go from figures. I can divide the second figure in half. Then 2 out of 8 pieces will be colored. In the first figure, I combine triangles as well.*	*…* *S: Actually, if we put the blue pieces together, there comes the same image as the figure below. So, it is equal.*  *A: Do you think there is an alternative solution to this problem?* *S: There certainly is, but I can't think of it.*
Operation Form		
Ali (NSB)	**İnci (RB)**	**Su (NSB)**
The answer is C … 4/16 is of the same magnitude as 1/4. ***Both are quarters.***	*İ: I think it's C. If we simplify 4/16, we get 1/4, or if we expand 1/4, we get 4/16.* *A: So how can you decide without simplifying or expanding?* *İ: No decision can be made without simplification or expansion.*	*S: I think 4/16 is smaller. If I were to give an example of cake again, the size of my slice would be smaller, so the answer is B.* *A: Can it be solved in a different way?* *S: If there is an operation for it, it can be done, but I can't do it because I don't remember.*
Scenario Form		
Ali (NSB)	**İnci (RB)**	**Su (EE)**
… it cannot be divided into 1 piece. It would be pointless to divide it into 1 piece, because then it would be a whole, and we would not have divided it. Then, they should divide it into 4 pieces and eat ***1 slice****, so both of them will eat ¼. They both eat a quarter cake.*	***İ: I'm sure only of the first blank****. I'm not sure of the others.* ***It says the same magnitude****.* ***If Ahmet ate 4 pieces, Ali should also eat 4 pieces. Both eat the same amount after all****. So* ***4/16 for Ahmet. And it's 1/4****. If we think of multiplying what by what results in 1/4, then comes 1. 1/4 = 1/4. It's correct, then.*	*S: Ahmet eats 4/16, and Ali eats* ***1/4 quarter****.* *A: So how much of the cake did the two of them eat?* *S: Half? I thought they're 2 friends. If one gets one, the other gets one, too.*

Table 10 shows that Su and Ali used NSB solution strategy in figure form, and Ali supported the correctness of the problem he solved with RB solution strategy. Another remarkable result in the context of figure form is for İnci. The dialogue between İnci and the researcher suggests that she memorized obtaining equivalent fractions as a rule, but she did not know what it means. On the other hand, when an alternative solution to the question was requested, the student resorted to NSB solution strategy. However, she was unaware that her NSB answer was the explanation of her RB solution. She could not relate the two answers to each other.

Another noteworthy result in Table 9 is that the question with the highest number of mistakes is in scenario form. Table 10 suggests that the reason why the students made mistakes in scenario form was that they had difficulty in associating the given fractions with verbal situations. The examination of İnci's answer indicates that she was aware that $\frac{1}{4}$ and $\frac{4}{16}$ were equal, but she could not complete the scenario correctly. Su, on the other hand, gave an answer completely separate from the scenario. In addition, it can be said that the students considered the situation of eating equal amounts of two cakes divided into different slices as the equality of the number of pieces.

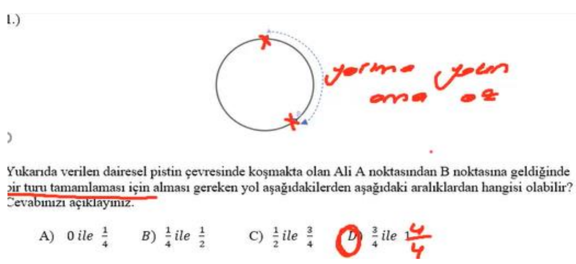
c) Flexible computing and deciding the reasonableness of the result

Table 11 below presents the solution strategies used by the students while answering the questions prepared in different forms for addition and subtraction with fractions in the component of "flexible computing and deciding the reasonableness of the result".

Table 11. Students' Solution Strategies in the Component of Flexible Computing and Deciding the Reasonableness of the Result

		Figure Form		Operation Form		Scenario Form		
		Correct	Wrong	Correct	Wrong	Correct	Wrong	Blank
Subtraction	Ali (HA)	NSB		PNSB		PNSB		
	İnci (MA)		NSB	RB		RB		
	Su (LA)		HA*		RB			X
Addition	Ali (HA)	NSB		NSB		NSB		
	İnci (MA)	NSB		NSB		RB		
	Su (LA)	NSB			RB		HA*	

Table 11 shows that the students mostly used NSB solution strategy in questions in figure form. The biggest number of wrong answers was in the subtraction question in figure form. In order to see the reason for this difference, İnci's answers for subtraction (Figure 1) and addition (Figure 2) are presented as she applied NSB solution in both questions in figure form.



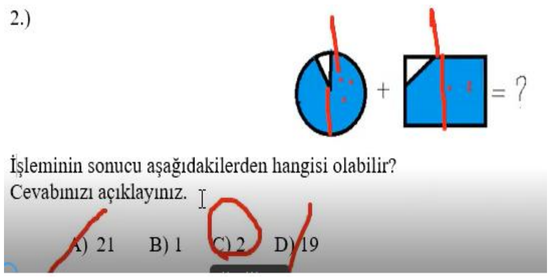


Figure 1. İnci's solution of the subtraction question in figure form **Figure 2.** İnci's solution of the addition question in figure for

In the interviews, İnci explained the solution she applied in Figure 1 as follows: "*The distance he covered is close to half, but less than half. Then the distance he will cover should be more than half. I made it 4/4 instead of 1. That's why he covers the most distance there, so I said D.*" In Figure 2, she said: "*I think it's 2. Because both of them are close to whole again. If we considered them as whole, there would be 2 wholes*". The solutions and the explanations of the solutions suggest that İnci first made NSB inference. However, while the answer to the question was equal to an integer in the addition operation, the range of the correct answer was asked in the subtraction operation. Although İnci was aware that the result would be larger than half, she could not decide on the most appropriate interval.

Another striking finding in Table 11 is that the students tended to use NSB solution strategy more in the addition question in operation form. This is most probably because the addition question cannot be easily solved by equating the denominators. In that question, both Ali and İnci first thought of equating the denominators, but both the statement "find the answer without performing any operation" in the root of the question and the fact that the denominators could not be easily equated directed the students to use NSB solution strategy. In addition, when the question forms given for the component of performing flexible operations and deciding the reasonableness of the result were examined, it was seen that the students tended to think whether the answer given was reasonable by comparing it with the whole text, especially in scenario form. After the students gave their initial answers, they read the whole text and changed the answers that disrupted the integrity of the meaning in the text. This result was observed in all questions in scenario form used in the study.

d) Understanding the meaning and effect of operations

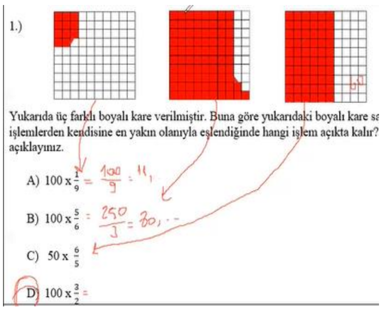
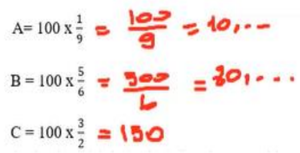
For the component of "understanding the meaning and effect of operations", the students were asked questions in different forms in which they had to decide how the result would change when a natural number was multiplied by a fraction larger or smaller than one. Table 12 presents the solution strategies used by the students while answering these questions.

Table 12. Students' Solution Strategies in the Component of Understanding the Meaning and Effect of Operations

	Figure Form		Operation Form			Scenario Form	
	Correct	Wrong	Correct	Wrong	Blank	Correct	Wrong
Ali (HA)	NSB		PNSB			NSB	
İnci (MA)	RB		RB				RB
Su (LA)		EE			X	NSB	

Table 12 shows that in the question in operation form, Ali (HA) used PNSB and İnci (MA) used RB solution strategy, while Su (LA) left the question blank. On the other hand, two out of three students used NSB solution strategy in scenario form. Another noteworthy result in Table 13 is that the only correct answer of Su (LA) was to the question in scenario form, which she solved with NSB solution strategy. To explore the reason for such difference between the solution strategies used by the students in the questions given in different forms, the students' explanations regarding the question are given in Table 13.

Table 13. Student Answers for the Component of Understanding the Meaning and Effect of Operations

<table>
<tr><th colspan="3">Figure Form</th></tr>
<tr><th>Ali (NSB)</th><th>İnci (RB)</th><th>Su (EE)</th></tr>
<tr><td>In the first figure, about 11 out of 100 are colored. The closest to this is 1/9 because if it was 1/10, it would be 10, so there is not much difference. In the second figure, there are around 83. Let's say 80. There should come out a number very close to 100. If we multiply by 6/6, we get 100. If we multiply by 5/6, we get a number close to 80. Then there's the compound fraction multiplied by 50. The answer will be between 50 and 100. The answer would probably be close to 60. The last choice isn't possible anyway because it would be more than 100.</td><td>I just multiplied them all.

</td><td>I'm actually torn between C and D. In both, the top is larger than the bottom (compound fraction). But all options have 100, and C has 50. I say C.</td></tr>
<tr><th colspan="3">Operation Form</th></tr>
<tr><th>Ali (PNSB)</th><th>İnci (RB)</th><th>Su (Blank)</th></tr>
<tr><td>Since all three letters in choice A are multiplied by 100, when we order the fractions from smallest to largest, we also order the results of the operations. If we equate the denominators, we get C>B>A.
In choice B, for it to be larger than 100, it must be multiplied by a number larger than one. It can't be choice B as well.
I need to perform an operation for choice C. 100:2=50x3=150 It can't be C.
In choice D, the value of A is smaller than 25 because I need to find 100x1/9. If we found 1/10 of it, it would make 10, that is, approximately 11. The answer is D.</td><td>I calculated each choice one by one. In this case, choice D is wrong. The value of A is not larger than 25.

</td><td>(after 3 minutes of deliberation) I don't know, I can't answer this question.</td></tr>
<tr><th colspan="3">Scenario Form</th></tr>
<tr><th>Ali (NSB)</th><th>İnci (RB)</th><th>Su (NSB)</th></tr>
<tr><td>Now the first blank, that is her height will be 100 cm, I'm sure of it. Because there can't be a height of 5/6 cm. It could be if it said meters. I eliminated 3/2 because getting shorter by 3/2 means getting taller anyway. Why would it say she got shorter then? It would be meaningless. The height should be shortened more in the first blank than in the second. In other words, the number I will put in the first must be smaller than the number I will put in the second. If it was 1/9 of it, it would mean about 11 cm in height. If she got taller by 5/6 then, she could not reach her original height, but it would mean getting taller.</td><td>The 100 cm height would be 3/2 because 100 is divisible by 2. The second one can be 5/6 because the denominator of 3/2 can be 6… If we perform the operations, there comes 100.3/2=150, but that doesn't make sense. The height needs to be shortened, but it gets taller here. It can't be like this, I have to change it, but other numbers are not multiplied somehow… I think it should be an integer, but it isn't. … I don't know, whichever I multiply by, it's not an integer. I'm confused.</td><td>The first blank should be 100 because a person cannot be 3/2 cm tall. I cannot write 3/2 in the second blank because the bottom number is smaller than the top one… Since it must be smaller, 1/9 should come in the second blank. I said 3/2 for the last blank, but if it is 3/2, it will exceed her former height. I thought it would exceed the former height, but it didn't, so it should have been 5/6. So, if it was just a little more, it would be 6/6. Then it would be the same height as before.</td></tr>
</table>

The student answers in Table 13 show that in figure form, Ali solved the question with NSB solution strategy without performing any operation, while İnci used RB solution strategy by finding the results of the operations given in each choice and matching them with the figures. Su, on the other hand, answered by considering the situation that was different from the ones in other choices without going into any mathematical thinking process.

The student answers in operation form indicate that Ali interpreted some choices based on NSB solution strategy and some based on RB solution strategy, while İnci found the results of all operations and compared them with the choices. On the other hand, Su first thought of departing from the choices, but she could not make any comments on the question.

As shown in Table 13, the students mostly used NSB solution strategy in the question in scenario form. In addition, Su (LA) solved only the question in scenario form correctly and by using NSB solution strategy. The student answers in scenario form in Table 13 suggest that Ali and Su answered the question correctly, thinking that the multiplication of a natural number by a quantity larger than an integer would bring a number larger than it. On the other hand, İnci, thinking that the height should be an integer, took into account the situations in the choices where an integer would come out when multiplied by 100; however, even though she was aware of the lack of semantic integrity when she completed the text, she could not find an alternative solution. Also, the fact that all three students thought that the height should be an integer contributed to their reaching the correct result.

e) Measurement Benchmarks

Table 14 below presents the solution strategies used by the students in the questions in different forms in which they were expected to make a decision about the whole by referring to a part for division with fractions in the component of "measurement references".

Table 14. Students' Solution Strategies for the Component of Measurement References

	Figure Form		**Operation Form**		**Scenario Form**	
	Correct	**Wrong**	**Correct**	**Wrong**	**Correct**	**Wrong**
Ali (HA)	PNSB		RB		RB	
İnci(MA)		PNSB	RB			RB
Su (LA)		NSB		RB		EE*

Table 14 shows that the students mostly used RB solution strategy in operation and scenario forms and PNSB solution strategy in figure form. The students used RB solution strategy more in the questions in different forms prepared for measurement references than other number sense components. This is most probably because the question is about division with fractions. To examine these results in depth, the answers given by the students to each question form are given in Table 15.

Table 15. Student Answers to Questions in Different Forms for the Component of Measurement Benchmarks

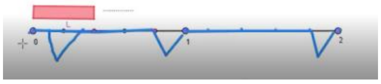
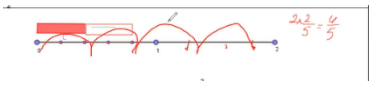
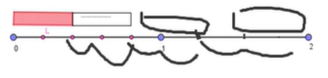
Figure Form		
Ali (PNSB)	**İnci (PNSB)**	**Su (NSB)**
… If 2 cm by 2/5 cm tapes are to be put, I make it 2:2/5. If we flip it upside-down and multiply, we get 5. Or we can think over the figure. 2 full and 1 half pieces fit in each interval. If we combine the rest, we get 5 again. 	*At first, I made 2.2/5 and found 4/5… It could be something like this. I need to put 3 dots between 1 and 2 for all of them to be 5. If we put them and order by 2, it makes 4. Actually, it's not exactly four. Because it doesn't reach exactly 2. But I still say A.* 	*Two gone in the first tape, so let's do it that way … If I put something like this myself (putting dots). I can draw another tape, there is still distance. We found 4 tapes, could the answer be 4/5? There are 5 in the denominator but nothing in the other.* 
Operation Form		
Ali (RB)	**İnci (RB)**	**Su (RB)**
The answer to this question can be found, but it's very challenging without operation… Dividing a number by a fractional number means multiplying that number by the inverse of this number. So, if I multiply by the denominator, it will be 10. If I divide it by 2, it will be 5. 2x5/2= 5.	*İ: I actually did this by performing an operation, but the question requests doing it without an operation.* *A: How did you do it?* *İ: I flipped the second one upside down and multiplied it. 2. 5/2, that's 5. I think this question cannot be solved without operation. This is multiplication and division.* ***How to do it without operation?***	*It says 2:2/5. I say the answer is 1/5. If I divide 2 by 2, it will be 1.*
Scenario Form		
Ali (RB)	**İnci (RB)**	**Su (EE)**
… *If she took 1/5 l of water and used it 2 times, it would be (1/5+1/5=2/5). I'm sure it will be surely like this. Well, she would water it for 5 days with a 2-l water bottle. Because 2: 2/5, that is 10/5: 2/5, which would make 5 days.*	*10 l of water is taken, and Ayşe fills the bowl twice a day. If we write 1/5 daily, it becomes 1/5 of 10 l. So, it becomes 2, and 2 l of water. If she used 2 l of water a day, how many days would it take for it to be finished, I thought. It would be finished in 5 days.* *(reading again) but it seems illogical. I actually departed from the choices.* *10.1/5= 2 l* *10: 2 = 5 days.*	*S: If she used 1/5 l of water … If the bowl took 10 l… If she used 1/5 l a day… she would use it for 5 days.* *A: So how did you fill in the blanks? Can you share with me?* *S: I'm confused actually. I just think so.*

Table 15 shows that only Ali answered the question in figure form correctly, he first used RB solution strategy and explained the correctness of his solution using NSB solution strategy. Therefore, it can be said that Ali used PNSB solution strategy. The answers of Su and İnci indicate that both students drew tapes on the figure, but they could not express the number of tapes. İnci also used RB solution strategy, but her operation was not for the solution of the problem.

In the question in operation form, all three students used RB solution strategy. However, Su answered the question incorrectly because she could not remember the rule. Ali and İnci, on the other hand, stated that they solved the question based on the rule "write the first fraction exactly as it is, flip the second fraction upside-down and multiply". In addition, İnci's statement that "*multiplication and division cannot be done without operations*" is also remarkable. In other words, the students tended to flip and multiply without thinking about what division means.

Similarly, in scenario form, Ali and İnci headed towards RB solution strategy. On the other hand, İnci's answers suggest that she performed operations with the numbers given in the choices and tried to find a number in the choices as a result. However, she could not complete the scenario correctly, though she realized that the integrity of meaning in the text was not achieved when she filled in the blanks.

Discussion and Conclusion

This study was conducted to determine the number sense strategies used by eighth-grade students with different academic achievement levels in fractions questions in figure, operation, and scenario forms. To this end, the study determined the students' uses of number sense in the questions in figure, operation, and scenario forms, and the number sense components they used in the questions given in different forms were examined separately.

The students used number sense-based solution strategy in the questions given in figure form the most, which were followed by scenario questions and operation questions, respectively. The structure of the curriculum can be cited as one of the reasons why the students resorted to number sense-based solutions more in the questions given in figure form. The mathematics curriculum (Ministry of National Education [MoNE], 2018) suggests using various models such as fraction blocks, pattern blocks, and number lines in teaching fractions. The use of such visual representations may have supported the development of number sense in students. The students may have made inferences based on figures, as visual representations are effective in improving students' skills of making generalizations and adapting strategies to concepts (Kamii, Kirkland, & Lewis, 2001) and contribute to their intuitive perceptions (Usiskin, 1987). In addition, the obtained result is consistent with the studies in the literature emphasizing that the use of number sense is much more common when students deal with situations involving visual representations (Can & Yetkin Özdemir, 2020; Kayhan Altay, 2010; Yapıcı, 2013). The evaluation of the students' answers in terms of academic achievement showed that the student with high academic achievement applied number sense-based solution strategy more. On the other hand, the student with low achievement used number sense-based strategy more in the questions in figure and scenario forms compared to the student with medium achievement. This result is parallel to the result of Can (2019) but is different from the results of some studies (Harç, 2010; Yang, 2005; Yang et al., 2008). Such difference may be due to the structure of the question forms used in the studies. This is because the previous studies reporting a positive relationship between academic achievement and number sense have mainly covered questions in operation form, whereas Can (2019) addressed questions in the form of context problems. From this point of view, it can be said that this result of the present study is parallel to the literature.

The student with low academic achievement only correctly answered the questions in figure and scenario forms, which she answered with number sense-based solution strategy. She answered all questions in operation form incorrectly. The reason why all students mostly resorted to rule-based solution for the question in operation form may be because teaching environments mostly involve questions in operation form and these questions are solved based on algorithms. Insufficient questioning of students' solutions may lead a group of students who do not apply algorithms correctly, forget the rules, or remember them wrongly to be considered unsuccessful in mathematics (Can, 2019). As a matter of fact, the literature contains studies emphasizing that students who have no procedural knowledge on the subject can answer questions correctly based on number sense (Clarke & Roche, 2009; Riddle & Rodzwell, 2000). In this context, students' number sense skills can be used as a means of supporting their conceptual learning in appropriate question forms.

Another remarkable result of the study is that the question in which the students made the most mistakes is in scenario form. This can be explained by the fact that the students were not accustomed to such questions and by their unfamiliarity with the context. In addition, only conceptual or procedural knowledge may not be sufficient to answer questions in scenario form. This is because questions in scenario form also require reading and comprehension skills, and these skills play an important role in problem solving. As a matter of fact, many studies in the literature show that there are significant relationships between reading comprehension and mathematics achievement (Gersten, Jordan, & Flojo, 2005; Kyttala & Björn, 2014; Louange, 2007) and number sense (Can & Yetkin Özdemir, 2020). These results obtained from the study support the mentioned studies.

It was observed that the students mostly used rule-based solution strategy in the questions in operation form. The studies in the literature also mainly used questions in operation form, and as a result, determined that students and pre-service teachers used rule-based methods in such questions (Der-Ching & Hung-Jin, 2019; İymen & Duatepe-Paksu, 2015; Markovits & Sowder, 1994; Şengül & Gülbağcı Dede, 2014; Yang et al., 2009; Yang & Huang, 2004; Yenilmez & Yıldız, 2018). This result of the study also shows parallelism with the mentioned studies.

The evaluation based on number sense components revealed that the students first tended to apply the "denominator equalization" method, a rule-based solution strategy, in the questions involving comparing fractions in different forms for the component of "understanding of the meaning and size of numbers". However, since the questions in operation and figure forms given in the present study could not be solved only by "denominator equalization", the students resorted to number sense-based solution strategy. Similarly, the literature highlights that in question forms where it is not easy to apply standard algorithms, students try to find non-standard solutions (Markovits & Sowder, 1994) and tend to use number sense (İymen, 2012).

Another result for the component of understanding of the meaning and size of numbers is that the students tended to check the closeness or distance of fractions to half. On the other hand, when the students could not interpret these distances, they again resorted to rule-based strategies. One of the underlying reasons for this may be the lack of emphasis on such strategies in the classroom environment. As a matter of fact, when Şengül and Gülbağcı Dede (2014) asked a similar question to postgraduate teachers, they found that only 2 teachers could solve the question based on number sense. The results reported by Der-Ching and Hung-Jin (2019) are also similar. Hence, it can be said that apart from the effect of different assessment tools on students' use of number sense, the effects of teachers are too obvious to be ignored. Another noteworthy result about the use of number sense in the comparison of fractions is that the student with low academic achievement made comparisons by looking at the difference between the numerator and the denominator of two fractions. This shows that the student ignored the magnitude of the denominator and therefore the size of the relevant parts (numerator/denominator ratio). For this reason, it can be said that the student had an integer thinking style in which she considered the absolute difference between the numerator and the denominator. Similarly, Clarke and Roche (2009) emphasized that sixth-grade students compared fractions by focusing on the difference between the numerator and the denominator of the fractions 5/6 and 7/8, while Markovits and Sowder (1994) stated that eighth-grade students thought that the fractions 5/6 and 9/10 were equal and they were in one piece distance to whole. Therefore, it can be said that this result of the study is consistent with the literature.

One of the interesting results for the component of "understanding and use of equivalent expressions" is that the student with low academic achievement could determine the equality of two fractions by using number sense in the question in figure form, although she did not know how to obtain an equivalent fraction. Another result is that the student with medium academic achievement could determine the equality of two fractions in a rule-based way, but could not explain why they are equal. On the other hand, when the same student was asked to find an alternative solution, she divided one of the figures in half and said that the number of pieces and the colored areas were equal. Both results clearly show that the use of number sense serves as a bridge between students' intuitive thinking and the construction of conceptual knowledge. Likewise, many studies in the literature suggest that procedural skills are not sufficient for meaningful learning (Barnes, 2020; Burns, 1994; Hiebert, 1999; NCTM, 2000; Yang & Huang, 2004) and that number sense allows establishing connections between concepts, seeing number patterns, and detecting numerical errors, thus leading to meaningful learning (Şengül, 2013). Additionally, for the component of "understanding and use of equivalent expressions", the students mostly answered the question in scenario form incorrectly. One reason for this is that the students had difficulty in associating fractions with verbal situations, and the other is that they did not know what the numerator and the denominator in fractions mean. Considering the students' mistakes, it can be said that their misconceptions about fractions caused them to make mistakes. The literature also contains studies emphasizing that students' misconceptions affect the use of number sense negatively (Clarke & Roche, 2009).

For the component of "flexible computing and deciding the reasonableness of the result", two questions based on addition and subtraction with fractions were asked to the students. In all questions given in scenario form, the students took into consideration whether the result was appropriate or not. This was observed in all questions in scenario form through the experiments, regardless of number sense components. This result shows parallelism with the studies in the literature that emphasize that the ability to decide whether the result is reasonable or not will improve through scenario completion problems (Greenes et al., 1993; Yang, 2006). Therefore, it can be said that scenario form is important to the development of students' number sense.

The students used number sense-based solution strategy more in the addition question in operation form compared to the subtraction question. This is because there are larger numbers in the addition question, and it takes more time to equalize the denominators of these numbers compared to the subtraction question. For this reason, the students sought an alternative solution instead of equating the denominators. This result shows parallelism with the result of the studies exploring students' number sense development, which suggest that the tendency to use algorithms is higher in small numbers (Markovits & Sowder, 1994; Wearne & Hiebert, 1988; Yang & Hsu, 2009). Therefore, it may be effective for students to work with larger and more challenging numbers for their number sense skills to improve.

For the component of "understanding the meaning and effect of operations", the students tended to provide rule-based solutions in figure and operation forms, while they used number sense much more in scenario form. In a similar vein, the studies in the literature conducted with different sample groups and using questions in operation form concluded that the use of number sense is very low in the component of "understanding the meaning and effect of operations" (Harç, 2010; Zanzali & Ghazali, 1999). Thus, fill-in-the-blank exercises in the scenario form may be included in course contents, especially in the component of "understanding the meaning and effect of operations".

In the component of "measurement benchmarks", the students mainly used rule-based solution strategy. Similarly, Reys and Yang (1998) determined that all students with medium academic achievement used rule-based solution strategy in the division-based question asked for the component of "measurement benchmarks". The main reason for the similarity between the result obtained by Reys and Yang (1998) and that of the present study may be about the subject area of the question asked. Especially in fraction division, many students perform rule-based operations based on flipping and multiplying the divisor without thinking about the meaning of division (Tirosh, 2000). In this context, it is considered important to associate concept teaching with students' intuitive solutions based on number sense.

Recommendations

Based on the results obtained in the study, the following recommendations are presented for both practitioners and future studies:

- This is a qualitative study, and the results of the study are limited to three students. Hence, it is important to conduct a study in which students' number sense skills are examined with a larger study group in terms of the generalizability of the results.
- While the international literature contains studies examining the number sense of students through scenario completion activities, the national literature includes no such study in the context of Türkiye, as literature review shows. This gap in the literature can be eliminated by conducting studies exploring the use of number sense by students or pre-service teachers in the questions in scenario form in different subject areas.
- The study group of this study consisted of students with different academic achievements, and it was found that the number sense performances of the students differed in operation form but were similar in scenario and figure forms by student achievement level. Therefore, it may be suggested to study with a larger sample group to examine whether there is a significant relationship between students' achievement levels and the improvement of number sense by the assessment tool used.
- Questions in different forms reveal different number sense strategies of students. For this reason, it can be said that it would be beneficial for teachers who want to improve students' number sense skills to use assessment tools containing questions in different forms. While creating assessment tools, care should be taken to use large numbers that make performing operations difficult and do not lead students to find a definite answer.
- It is recommended that teachers who aim to improve number sense skills in students for the component of performing flexible operations and deciding the reasonableness of the result work with questions in scenario form.
- The use of number sense acts as a bridge between students' intuitive thinking and the construction of conceptual knowledge. Therefore, conceptual knowledge about fractions can be built on students' number sense-based solutions.

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Appendix-1

Number Sense Test

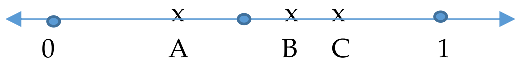
Figure Form	**Operation Form**	**Scenario Form**

Achievement: Compares, orders, and displays fractions on the number line

Target Number Sense Component: Understanding of the meaning and size of numbers

Figure Form

F-**1**



Which of the following numbers is most appropriate to write in the places indicated by the letters A, B, and C on the number line, respectively? Explain.

$A)\ \frac{4}{9};\frac{3}{4};\frac{8}{9}$ $B)\ \frac{2}{3};\frac{2}{5};\frac{4}{7}$

C) $\frac{1}{4};\frac{4}{7};\frac{3}{8}$ D) $\frac{2}{5};\frac{4}{7};\frac{2}{3}$

Operation Form

O-1 $\frac{2}{3};\frac{2}{5};\frac{4}{7}$

Which of the following statements about the fractions given above is correct? Explain.

$A)$ The fraction $\frac{2}{3}$ is smaller than $\frac{2}{5}$.

$B)$ The fraction $\frac{2}{5}$ is closer to whole than the fraction $\frac{4}{7}$.

$C)$The fraction $\frac{2}{5}$ is closer to half than the fraction $\frac{2}{3}$.

D) The descending order of the given fractions is $\frac{2}{3},\frac{2}{5},\frac{4}{7}$.

Scenario Form

S-1 "Lucky Luke" is a comic book drawn by Belgian cartoonist Morris and has been adapted as a cartoon in our country. In the cartoon, Lucky Luke, who is a lone cowboy, catches the Dalton Brothers, who are involved in a different crime in each episode, together with his faithful horse Jolly Jumper. Averell is the tallest and the most obsessed with eating. The wisest and the shortest one, Joe is …2/5…. of Averell's height. William is taller than Joe but is … 4/7… of Averell's height. Jack, on the other hand, is taller than William, but he is …2/5 … of Averell's height.

Complete the given story by filling in the blanks with the fractions given below appropriately.

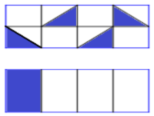
$\frac{3}{2}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{4}{7}$

Achievement: Compares, orders, and displays fractions on the number line

Target Number Sense Component: Understanding and use of equivalent expressions

Figure Form

F-**2**



Choose the appropriate one from among the statements given below for the two fractions shown with the model in the figure. Explain your answer.

A) The fraction modeled in figure I is $\frac{4}{12}$.

B) The fraction modeled in Figure II is larger.

C) The fractions given in both figures are not equal.

D) The fractions given in both figures are equal.

Operation Form

O-2 For the fractions $\frac{4}{16}$ and $\frac{1}{4}$, choose the appropriate one from among the statements given below. Explain your answer.

$A)$ The fraction $\frac{4}{16}$ is larger than the fraction $\frac{1}{4}$.

$B)$ The fraction $\frac{1}{4}$ is larger than the fraction $\frac{4}{16}$.

C) The fractions are equal.

Scenario Form

S-2 Ahmet and Ali attended their friend's birthday party. On the birthday, there were two cakes of the same size, one with chocolate and the other with fruit. Because Ahmet loves chocolate, he ate chocolate cake, while Ali ate fruit cake. After the cakes were eaten, the following conversation took place between Ali and Ahmet.

Ahmet: They cut the chocolate cake into ..16.. pieces, but I ate 4 pieces.

Ali: They cut fruit cake into …4… pieces, I ate …1… slice.

Ahmet: I think we both ate the same amount of cake.

F-5

D) The fractions are not equal.

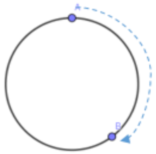
Ali: No, I don't think so. You ate 4 pieces.
Ahmet: Oh! You smart! The amount we both ate is actually 1/4 of the cake.

Fill in the blanks and complete the dialogue.

16 1 ¼ 4 ½

Achievement: Performs addition and subtraction with fractions.
Target Number Sense Component: Performing flexible operations and deciding the reasonableness of the result

F-3



What could be the range of the distance that Ali who is running around the circular track given above should cover to complete a lap when he arrives in point B from point A?

$0\ to\ \frac{1}{4}$ $B)\ \frac{1}{4}\ to\ \frac{1}{2}$

C) $\frac{1}{2}\ to\ \frac{3}{4}$ $D)\ \frac{3}{4}\ to\ 1$

O-3

$$1-\frac{2}{5}$$

What is the range of the result of the operation above? Explain.

$A)\ 0\ to\ \frac{1}{4}$

$B)\ \frac{1}{4}\ to\ \frac{1}{2}$

C) $\frac{1}{2}\ to\ \frac{3}{4}$

$D)\ \frac{3}{4}\ to\ 1$

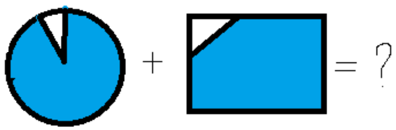
S-3 **Around the World in Eighty Days**

Mr. Fogg is famous for his wealth, politeness, meticulousness, and punctuality. One day, at the "Reform Club", of which he is a member, he makes a bet on a seemingly impossible matter, the wager being for half of his fortune, and says that he will circumnavigate the world in 80 days. Mr. Fogg embarks on this journey the very next day, as a single delay or a single mishap would cause him to lose everything. He first reaches from London to Africa in 7 days, from Africa to India in 10 days, and from India to Japan in 15 days. When Mr. Fogg arrives in Japan, he has used …2/5… of the time he is supposed to have on the trip. There remains …3/5… of the time for Mr. Fogg to return to London without losing the bet. Accordingly, the time remaining for Mr. Fogg to complete the circumference is between …1/2… and ¾…

Complete the story by filling in the blanks in the most appropriate way.

$\frac{3}{5}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{1}{4}$

F-4



Which of the following could be the result of the operation above?

$\frac{7}{8}+\frac{12}{13}$ O-4

Mark the answer closest to the result of the above operation without performing any operation. Explain your answer. (NAEP, 1978, as cited in Carpenter et

S-4 Although mathematics and music are thought to be concepts that are very far from each other, a mathematician named Pythagoras discovered the notes in music. One day, while Pythagoras was passing by a blacksmith workshop, he noticed that the sounds made by the blacksmith changed depending on the tools he used. Thereupon, Pythagoras made the blacksmith use various tools, examined the sounds, and took notes.

21 B) 1 C) 2 D) 19

al.,1980; NAEP, 2014, as cited in Lortie et al., 2015.)

21 B) 1 C)2 D) 19

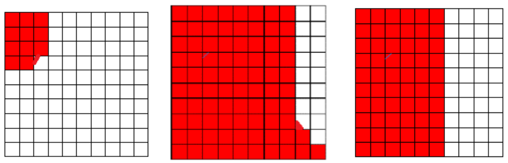
According to these notes, the length of the string C is longer than the string lengths used for the notes B and A. While …7/8... of the length of a string that makes C sound gives the sound of B, …12/13… of it makes A sound, and …2… of it makes D sound. The length of the string used for the note A is longer than that used for the note B. The length of the string used for the note D is almost as much as the sum of the lengths used for the notes B and A.

Identify the notes Pythagoras took by filling in the blanks appropriately.

7/8	12/13	19	2

Performs multiplication with fractions.

Target Number Sense Component: Understanding the meaning and effect of operations



Three different colored squares are given above. Which operation remains unmatched when the above colored square numbers are matched with the closest one of the following operations?

A) 100 x $\frac{1}{9}$

B) 100 x $\frac{5}{6}$

C) 50 x $\frac{6}{5}$

D) 100 x $\frac{3}{2}$

A= 100 x $\frac{1}{9}$ O-5

B = 100 x $\frac{5}{6}$

C = 100 x $\frac{3}{2}$

Which of the following is true regarding the results of the operations above?

The value of B is smaller than that of A.

The values of A and B are larger than 100.

The value of C is larger than 150.

The value of A is smaller than 25.

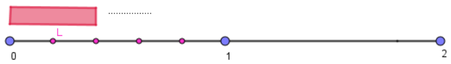
S-5 While Alice is sitting by the lake with her sister one day, Alice's attention is drawn to a rabbit with a clock that is talking and running around. Alice chasing this rabbit falls into a deep well just when she thinks she has caught the rabbit. As Alice explores the depths of the well, all she encounters is a glass table and a small door. Alice needs to be shorter in order to pass through that door. When Alice drinks the beverage on the table, her height of …100… cm becomes …1/9… of that height. However, this time, since she cannot reach the key, she has to be taller, and she drinks the beverage again and her height becomes …5/6… of her original height. Although Alice has not reached her original height, she cannot pass through the door and bursts into tears.

Fill in the blanks appropriately using the following.

100	3/2	1/9	5/6

Performs division with fractions.
Target Number Sense Component: Measurement benchmarks

F-6



On the number line with a length of 2 cm given in the figure, $\frac{2}{5}$ cm long tapes will be lined up side by side without leaving any spaces between them. How many pieces of tape are needed for this? Explain.

A) $\frac{4}{5}$ B) $\frac{1}{5}$ C) 5 D) 10

O-6

$$2 : \frac{2}{5}$$

Mark the answer closest to the result of the above operation without performing any operation. Explain your answer.

A) $\frac{4}{5}$ B) $\frac{1}{5}$ C) 5 D) 10

S-6

Water is life for animals. They also feel thirst, but they cannot tell us. Many projects have been launched to help stray animals, and water bowls have been distributed for them. Ayşe also places a water bowl in front of her house because she loves animals very much. Ayşe's water bowl takes …1/5… liters of water, and Ayşe fills the bowl twice a day. Using …2/5… liters of water per day, Ayşe fills the water bowl with a 2-liter water bottle for …5… days. When the water runs out, Ayşe refills the water bottle, and every day she reaches out to our friends on the street tirelessly.

Fill in the blanks appropriately using the following.

1/5	10	2/5	5